

# UIUC Mock Putnam Exam 2/2003

10-29-03

**Problem 1.** Evaluate

$$\sum_{n=0}^{\infty} (n+4)3^{-n}.$$

**Problem 2.** Find the volume of the solid region in space which is the intersection of the three open cylinders  $\{x^2 + y^2 < 1\}$ ,  $\{x^2 + z^2 < 1\}$  and  $\{y^2 + z^2 < 1\}$ .

**Problem 3.** Show (without using a calculator or doing extensive computation) that

$$\log_{2003} 2004 + \log_{2004} 2003 > 2$$

( $\log_a b$  denotes the base  $a$  logarithm of  $b$ ).

**Problem 4.** For  $a \geq 2$  evaluate the integral

$$\int_0^{\infty} \frac{1}{a^2 + (x - \frac{1}{x})^2} dx.$$