UIUC Mock Putnam Exam 3/2003
Elementary Version

11-19-03

Problem 1. Let

$$f(x) = \frac{1}{1-x}.$$

Let $f_1(x) = f(x)$ and for each $n = 2, 3, \ldots$, let $f_n(x) = f(f_{n-1}(x))$. What is the value of $f_{2003}(2003)$?

Problem 2. Given $x_0 = 0$, define $x_{k+1} = \frac{x_k^2 - 2}{2x_k - 3}$. Determine if the sequence (x_n) is convergent and if it is, find the limit.

Problem 3. Show that there is a multiple of 2003 which contains all ten digits.

Problem 4. Show that among any 101 points inside a square whose side has length 1, there exist two points whose distance is at most $\frac{\sqrt{2}}{10}$.

Problem 5. Let (a_n) be a bounded sequence of integers which satisfies the recurrence condition

$$a_n = \frac{a_{n-1} + a_{n-2} + a_{n-3}a_{n-4}}{a_{n-1}a_{n-2} + a_{n-3} + a_{n-4}}. \tag{1}$$

Show that the sequence is eventually periodic.