

# UIUC Mock Putnam Exam 1/2004

**Problem 1.** Let  $F_0 = 0$ ,  $F_1 = 1$ , and  $F_n = F_{n-1} + F_{n-2}$  for  $n \geq 2$ . Evaluate the series

$$\sum_{n=1}^{\infty} \frac{1}{F_n F_{n+2}}$$

**Problem 2.** Let  $\{c_n\}_{n \geq 1}$  be a sequence of positive numbers, and suppose that, for any sequence  $\{d_n\}_{n \geq 1}$  of positive numbers with  $\lim_{n \rightarrow \infty} d_n = 0$ , the series  $\sum_{n=1}^{\infty} c_n d_n$  converges. Show that the series  $\sum_{n=1}^{\infty} c_n$  converges as well.

**Problem 3.** Let  $a_1 = a_2 = a_3 = 1$ , and for  $n \geq 4$  define  $a_n$  recursively by

$$a_n = \frac{1 + a_{n-1}a_{n-2}}{a_{n-3}}.$$

Show that  $a_n$  is an integer for all  $n$ .

**Problem 4.**  $2n$  points are drawn on the circumference of a circle. In how many ways can these points be joined in pairs by  $n$  chords which do not intersect within the circle?