

# UIUC Mock Putnam Exam 3/2004

**Problem 1.** Let  $f(x) = x^3 e^{x^2} (1 - x^2)^{-1}$ . Find  $f^{(7)}(0)$ , the 7th derivative of  $f$  at 0. (No brute force calculations!)

**Problem 2.** Does there exist a multiple of 2004 whose decimal representation involves only a single digit? Explain!

**Problem 3.** Evaluate the sum  $\sum_{k=0}^n \binom{n}{k}^2 (-1)^k$ .

**Problem 4.** Let  $f$  be a real-valued continuously differentiable function on the interval  $[0, 1]$  with  $\int_0^1 f(x) dx = 0$ . Show that

$$\int_0^1 f(x)^2 dx \leq \frac{1}{2} \int_0^1 |f'(x)| dx \int_0^1 |f(x)| dx.$$