

UIUC Department of Mathematics

Mock Putnam Exam 1

September 29, 1997

1. Find all integer solutions of the equation $a^2 + b^2 + c^2 = a^2b^2$.
2. Show that the number $3^n + 4^n$ is divisible by 7 whenever n is an odd natural number.
3. Prove that if a pentagon (five-sided polygon) inscribed in a circle has equal angles, then its sides are equal.
4. Let $x_0 = 0$, $x_1 = 1$, and

$$x_{n+1} = \frac{x_n + nx_{n-1}}{n+1} \quad (n \geq 1).$$

Show that the sequence $\{x_n\}$ converges as $n \rightarrow \infty$ and determine its limit.

5. Evaluate the integral $\int_0^1 \ln x \ln(1-x) dx$. (You may use the formula $\sum_{n=1}^{\infty} n^{-2} = \pi^2/6$.)