UIUC Department of Mathematics

Mock Putnam Exam 2

October 13, 1997

1. Determine which of the two expressions $\sqrt{n}^{\sqrt{n+1}}$ and $\sqrt{n+1}^{\sqrt{n}}$ is bigger when n is an integer greater than 8.

2. The Fibonacci numbers F_n are defined by $F_0 = 0, F_1 = 1$ and $F_n = F_{n-1} + F_{n-2}$ for $n \geq 2$. Evaluate $\sum_{n=1}^{\infty} (F_n F_{n+2})^{-1}$.

3. Evaluate $\int_0^{\pi/2} \log(\sin x) dx$.

4. Prove that the expression

$$\sqrt{3 + \sqrt{3 + \sqrt{3 + \cdots}}}$$

converges (in an appropriate sense) and find its value.

5. Show that for every positive integer n the binomial coefficient $\binom{2n}{n}$ is divisible by $n+1$.