UIUC Mock Putnam Exam 2/2001

October 23, 2001

Name:

Rules: No calculators, books, notes, etc. Show all work, but do not hand in scratch work.

Grading: Problems will be graded in much the same way as the Putnam exam problems. Each problem is worth 10 points. Partial credit may be given, but only if significant progress towards a solutions is being made. As in the Putnam exam, a clear, logical write-up of the solution is essential. Graded exams will be returned in next Tuesday's training session. Exam solutions will be posted on the Math Contest Web Page (www.math.uiuc.edu/contests.html) within a few days.

Time: Two hours. If you do four problems, this works out to 30 minutes per problem, the same amount of time per problem you get in the Putnam exam (which has 12 problems, to be solved in 6 hours). The problems have not been grouped into elementary and advanced problems, but they are roughly arranged in increasing order of difficulty. **Circle those problems on the coversheet (this sheet) that you want to have graded.**

Scratch work: Do all scratch work on separate sheets and turn in only final, clean solutions. Cross out anything you do not want to be considered for grading.

1 You are standing at the ocean's edge awaiting the arrival of a sailing ship having a 60 foot high mast. When you first see the tip of the mast on the horizon, about how far away is the ship? You may assume the earth's radius to be 4000 miles and that your eyes are at height 7 feet above the water.

2 Show that $7^{1/3} + 9^{1/3} < 4$.

3 Find the sum of the infinite series

$$\sum_{n=1}^{\infty} \frac{1}{2n^2 - n}.$$

4 Suppose every point of the plane has been colored either red, or blue. Prove that, for one of these colors, there exist pairs of points at *every* mutual distance.

5 Let $f(n) = [n + \sqrt{n} + 1/2]$, where $[x]$ denotes the greatest integer $\leq x$. Determine, with proof, the set of positive integers m that can be expressed in the form $m = f(n)$ for some integer n.

6 Find the precise set of numbers x for which the series

$$\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{1}{(m + \sqrt{n})^x}$$

converges.