

UIUC Department of Mathematics

Mock Putnam Exam 3

October 27, 1997

1. Let $P(x) = \sum_{i=0}^n a_i x^i$ be a polynomial of degree $n \geq 1$ with real coefficients. Show that if $\sum_{i=0}^n a_i/(i+1) = 0$ then $P(x)$ has at least one real root.
2. Nine mathematicians meet at an international conference and discover that, among any three of them, at least two speak a common language, and that none of them speaks more than three languages. Prove that there are at least three mathematicians who can speak the same language.
3. Suppose each point in the plane is colored either orange or blue. Prove that there exists a color (either orange or blue) such that, for every positive distance d , there exists a pair of points of that color having distance d .
4. Let a_1, a_2, \dots, a_n be positive real numbers, and let b_1, b_2, \dots, b_n be a permutation of the a_i 's. Show that $\sum_{i=1}^n a_i/b_i \geq n$.
5. Show that $2^{-x} + 2^{-1/x} \leq 1$ for all positive real values of x .