UIUC Department of Mathematics

Mock Putnam Exam 3

November 9, 1998

Graded exams will be returned at next Monday's Putnam Training Session.

Solutions will be posted by the end of this week at
http://www.math.uiuc.edu/~hildebr/putnam/mockputnam.html.

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1. [Barbeau, #230] Evaluate $f(n) = 1^2 - 2^2 + 3^2 - \cdots + (2n-1)^2 - (2n)^2$.

2. Let F_n be the nth Fibonacci number, i.e., $F_1 = 1$, $F_2 = 1$, and $F_{n+1} = F_n + F_{n-1}$ for $n \geq 1$. Show that $F_n^2 + F_{n+1}^2 = F_{2n+1}$ for all $n \geq 1$.

3. [Putnam 1986] Determine the rightmost digit (in decimal) of $\left[\frac{10^{20000}}{10^{100}+3}\right]$.

4. [Putnam 1984] Express

$$\sum_{k=1}^{\infty} \frac{6^k}{(3^{k+1} - 2^{k+1})(3^k - 2^k)}$$

as a rational number.

5. [Newman, #87] Let $\{a_n\}$ be a sequence of positive real numbers satisfying $a_n < a_{n+1} + a_{n^2}$ for all n. Show that the series $\sum_{n=1}^{\infty} a_n$ diverges.

Sources:

[Barbeau] E. Barbeau et al., Five hundred mathematical challenges

[Newman] D.J. Newman, A problem seminar