

UIUC Department of Mathematics

Mock Putnam Exam 4

November 10, 1997

1. What is the largest possible value of the product of positive integers a_1, a_2, \dots, a_n , given that their sum is 1997?

2. In a convex n -gon ($n \geq 4$) all diagonals are drawn. How many intersection points in the interior of the n -gon (not counting vertices) are there if no three diagonals intersect at the same point?

3. Let a_1, a_2, \dots, a_n be real numbers with $\sum_{i=1}^n a_i = 1$. Prove that

$$\sum_{i=1}^n i a_i^2 > \frac{1}{2\sqrt{n}}.$$

4. (Putnam '95) Evaluate

$$\sqrt[8]{2207 - \frac{1}{2207 - \frac{1}{2207 - \dots}}}$$

5. Prove that in any party there exist two people that have the same number of friends present. (Assume that friendship is a symmetric relation: If A is among the friends of B , then B is among the friends of A .)