UIUC Department of Mathematics

Mock Putnam Exam 4

November 15, 1999

This exam is intended as a practice test for the real Putnam Exam and will be graded in the same way. To receive credit, you need to explain yourself clearly and succinctly; an answer alone won't do.

Graded exams will be returned at the next Putnam Training Session on Monday, 11/29. (Because of Thanksgiving, there will be no meeting on 11/22.)

Solutions will be posted by the end of this week at
http://www.math.uiuc.edu/~hildebr/putnam/mockputnam.html.

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Problem 1. Let z_1, z_2, z_3 be complex numbers satisfying (1) $z_1 z_2 z_3 = 1$ and (2) $z_1 + z_2 + z_3 = z_1^{-1} + z_2^{-1} + z_3^{-1}$. Prove that at least one of these numbers is 1.

Problem 2. Let S be a given sphere with center O and radius r. Let P be any point outside the sphere S, and let S' be the sphere with center P and radius PO. Let A denote the area of the surface of the part of S' that lies inside S. Prove that A is independent of the particular point P chosen.

Problem 3. Evaluate

$$\lim_{N\to\infty}\left(\frac{1}{2N+1}+\frac{1}{2N+4}+\frac{1}{2N+7}+\dots+\frac{1}{2N+3N+1}\right).$$

Problem 4. Show that for each positive integer n, $2n+1$ divides the binomial coefficient $\binom{2n+1}{n}$.

Problem 5. Find, with proof, the largest real number d such that, for any partition of a square of unit side into two sets, one of the sets contains two points with distance at least d.