

UIUC Department of Mathematics

Mock Putnam Exam 6

February 2, 1998

1. Find an explicit formula for the  $n$ -th term of the sequence

1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, ...

2. Determine (by an explicit formula) the number of subsets of an  $n$ -element set that have an *even* number of elements.

3. Determine the least value of  $x_1^2 + x_2^2 + \cdots + x_{10}^2$ , where  $x_1, x_2, \dots, x_{10}$  are real numbers satisfying  $\sum_{n=1}^{10} x_n \sqrt{n} = 1$ .

4. Evaluate the integral

$$I_n = \int_0^\pi \left( \frac{\sin(nx)}{\sin x} \right)^2 dx$$

for all positive integral values of  $n$ .

5. A car dealership that was open 7 days a week sold at least one car each day in 1997, and a total of 600 cars during that year. Prove that there was a period of consecutive days during which exactly 129 cars were sold. (Note that there were 365 days in 1997.)