

## Advanced Putnam Training Session 6: More number theory

1. (B1, Putnam 1988) Show that every composite integer  $n$  is expressible as  $n = xy + xz + yz + 1$ , with  $x, y, z$  positive integers.
2. For which pairs  $(n, m)$  of positive integers is  $\sqrt{n} + \sqrt{m}$  rational?
3. For which pairs  $(m, n)$  of integers  $\geq 2$  is  $\log_m n$  rational?
4. Show that all integers of the form  $n^4 + 4^n$ ,  $n = 2, 3, \dots$  are composite.
5. (A2, Putnam 1990) Is  $\sqrt{2}$  the limit of a sequence of numbers of the form  $\sqrt[3]{n} - \sqrt[3]{m}$ , where  $n$  and  $m$  are nonnegative integers?
6. Determine, with proof, the set of positive integers that can be expressed in the form  $m = [n + \sqrt{n} + 1/2]$  for some positive integer  $n$ .
7. Let  $a_n = [(2 + \sqrt{3})^n]$ . Show that  $a_n$  is odd for every nonnegative integer  $n$ .