

Problem Set 1: Binomial identities

1. $\sum_{k=0}^n \binom{n}{k}$
2. $\sum_{k=0}^n (-1)^k \binom{n}{k}$
3. $\sum_{k=0}^{2n} (-1)^k k^n \binom{2n}{k}$
4. $\sum_{k=0}^n \binom{n}{k}^2$
5. $\sum_{k=0}^n \frac{1}{k+1} \binom{n}{k}$
6. $\sum_{k=0}^r \binom{m}{k} \binom{n}{r-k}$
7. $\sum_{k=0}^n \binom{2k}{k} \binom{2n-2k}{n-k}$
8. $\sum_{k=0}^n \binom{n-k}{k} (-1)^k 2^{-2k}$
9. $\sum_{m=0}^n \binom{m}{k}$
10. $\sum_{k=0}^n \frac{\binom{m}{k}}{\binom{n}{k}} \quad (n \geq m)$

Problem Set 2: Combinatorial problems

1. How many subsets are there in a set with n elements?
2. How many of these subsets have an *even* number of elements?
3. How many ways are there to place an order of 10 donuts if there are 3 varieties to choose from?
4. In how many ways can n be written as a sum of k nonnegative integers, if the order is taken into account (so that, for example, $10 = 3 + 3 + 4$ and $10 = 3 + 4 + 3$ count as different representations)?
5. How many ways are there to form a committee of 5 in a group of 20 people?
6. How many 14 letter “words” can be formed by rearranging the letters of “FIGHTING ILLINI”.