Examples: Square Wave The square wave is defined by

$$f(x) = \begin{cases} 1 & x \in [0, \pi) \\ -1 & x \in (\pi, 2\pi] \end{cases}$$

The Fourier cofficients are given by

Here are some pictures of the first few terms of the Fourier series:
A couple of things to note

- It is not clear whether the series converges, although it can be shown that it does, by a fancy alternating series test.....
- Note that the convergence is not very uniform. Near the jump we have a big error.
- There is a demo of this at *http://www/falstad.com/fourier*

Sawtooth Wave The integral of the square wave is the "sawtooth" wave

$$g(x) =$$

The Fourier series for the sawtooth wave can be found by integrating termwise

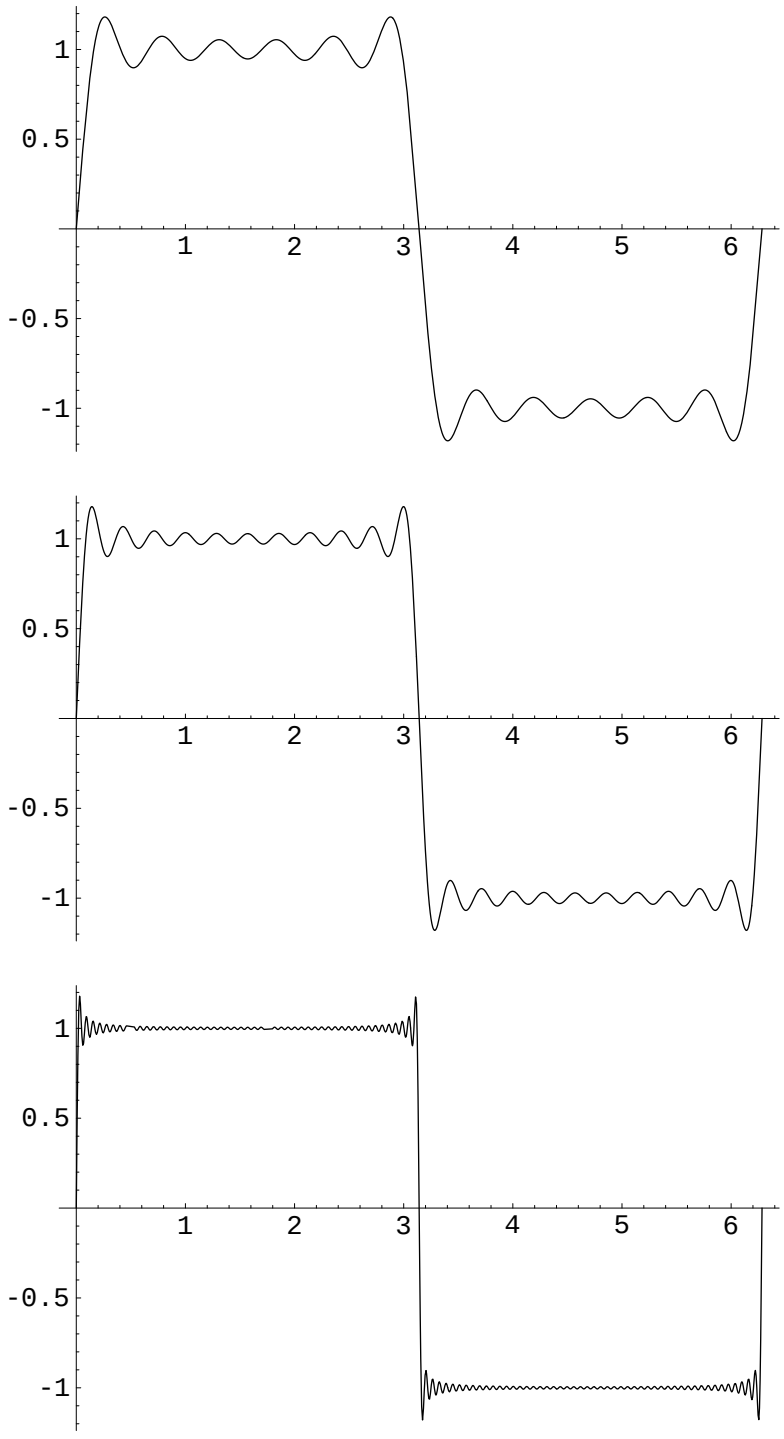

Figure 1: The Fourier series for the square wave with 11,21, and 101 terms.

the Fourier series for the square wave:

This Fourier series is convergent for all x by the comparison test.