**Practice Exercises for Lecture 4:**

Factor the following polynomials.

$$
\begin{aligned}
P(x) &= x^2 - 6x + 8 \\
P(x) &= 4x^2 - 16x + 15 \\
P(x) &= x^3 - 5x^2 + 6x \\
P(x) &= x^3 - 6x + 5
\end{aligned}
$$

**Hint:** In the last case note that $x = 1$ is a zero (root) of $x^3 - 6x + 5$. Use synthetic division to write $x^3 - 6x + 5 = (x-1)(x^2 + ax + b)$.

Find a partial fractions expansion for the following rational functions. The general form is given to you

$$
\begin{aligned}
\frac{7x-2}{x^2-6x+8} &= \frac{a}{x-2} + \frac{b}{x-4} \\
\frac{1}{4x^2-16x+15} &= \frac{a}{(x-3/2)} + \frac{b}{(x-5/2)} \\
\frac{x^2+x+1}{x^3-3x^2+2x} &= \frac{a}{x} + \frac{b}{(x-1)} + \frac{c}{(x-2)} \\
\frac{x^2-x+7}{x^3(x-1)}dx &= \frac{a}{x} + \frac{b}{x^2} + \frac{c}{x^3} + \frac{d}{x-1}
\end{aligned}
$$

Finish calculating the following integral, which was reduced using synthetic division in lecture

$$\int \frac{x^4 - 5x^2}{x^3 - 3x^2 + 2x} dx$$

Give the correct form of the partial fractions expansion for the following rational functions. Indicate if you need to do synthetic division and, if so, what the form of the result will be. You needn't actually calculate the relevant coefficients.

$$\frac{x^2-8}{x^3(x-1)(x-2)}$$

$$\frac{x^5-7x^3}{x^2(x^2-2x+1)}$$

$$\frac{x^3-1}{x(x^2-3x+2)}$$

$$
\begin{array}{r|l}
 & x^2 + x + 1 \\
\hline
x-1 & x^3 + 0x^2 - 6x + 5 \\
 & x^3 - x^2 \\
\hline
 & x^2 - 6x \\
 & x^2 - x \\
\hline
 & -5x + 5
\end{array}
$$

$-1 \pm \sqrt{\ }$

Practice Exercises: Lecture 4

$$x^2-6x+8 = (x-2)(x-4)$$

$$4x^2-16x+15 = (2x-3)(2x-5)$$

$$x^3-5x^2+6x = x(x^2-5x+6) = x(x-2)(x-3)$$

$$\begin{array}{r|l} & x^2+x-1 \\ \hline x-1 & x^3+0x^2-6x+5 \\ & x^3-x^2 \\ \hline & x^2-6x \\ & x^2-x \\ \hline & -5x+5 \end{array}$$

$x^2+x-1$ has roots

$$\frac{-1\pm\sqrt{5}}{2}$$

$$x^3-6x+5 = (x-1)\left(x-\left(\frac{-1+\sqrt{5}}{2}\right)\right)\left(x-\left(\frac{-1-\sqrt{5}}{2}\right)\right)$$

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$$\frac{7x-2}{x^2-6x+8} = \frac{A}{(x-2)} + \frac{B}{x-4}$$

$$A(x-4)+B(x-2) = 7x-2$$

$$A+B=7 \qquad -2A=12$$

$$-4A-2B=-2 \qquad \boxed{\begin{array}{l}A=-6\\B=13\end{array}}$$

$$= \frac{13}{x-4} - \frac{6}{x-2}$$

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$$\frac{1}{4x^2-16x+15} = \frac{A}{2x-3} + \frac{B}{2x-5}$$

$$A(2x-5)+B(2x-3) = 1$$

$$A+B=0 \Rightarrow B=-A$$

$$-5A-3B=1$$

$$-5A+3A=1$$

$$A=-\frac{1}{2}$$

$$B=\frac{1}{2}$$

$$= \frac{1}{2(2x-5)} - \frac{1}{2(2x-3)}$$

$$x^2+x+1 = a(x-1)(x-2) + bx(x-2) + cx(x-1)$$

@ $x=1$

$3 = -b \qquad b=-3$

@ $x=2$

$7 = 2c$

@ $x=0$ $\qquad a=\frac{1}{2} \quad c=\frac{7}{2} \quad b=-3$

$1 = 2a$

$$ax^2(x-1) + bx(x-1) + c(x-1) + dx^3 = x^2-x+7$$

@ $x=0 \qquad$ @ $x=1$

$-c = 7 \qquad d=7$

coefficient $x^3$

$a+d=0 \quad a=-d=-7$

coeff. $x^2$

$-a+b=1$

$b=1+a=\cancel{8}-6$

$$\frac{x^2-x+7}{x^3(x-1)} = \frac{-7}{x} + \frac{-6}{x^2} - \frac{7}{x^3} + \frac{7}{x-1}$$

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$$\begin{array}{r}
x+3 \\
x^3-3x^2+2x \;\big)\; x^4+0x^3-5x^2 \\
\underline{x^4-3x^3+2x^2} \\
3x^3-7x^2 \\
3x^3-9x^2+6x \\
2x^2-6x
\end{array}$$

$$\frac{x^4-5x^2}{x^3-3x^2+2x} = x+3 + \frac{2x^2-6x}{x^3-3x^2+2x}$$

$$= x + 3 + \frac{2x-6}{x^2-3x+2}$$

$$\frac{2x-6}{(x-1)(x-2)} = \frac{A}{x-1} + \frac{B}{(x-2)} = \frac{4}{x-1} - \frac{2}{x-2}$$

$$2x-6 = A(x-2) + B(x-1)$$

$$B = 2\cdot 2 - 6 = -2$$

$$-A = -4 \quad A = 4$$

$$\frac{x^4-5x^2}{x^3-3x^2+2x} = x + 3 + \frac{4}{x-1} - \frac{2}{x-2}$$

$$\int \frac{x^4-5x^2}{x^3-3x^2+2x}\,dx = \frac{x^2}{2} + 3x + 4\ln|x-1| - 2\ln|x-2| + C$$

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$$\frac{x^2-8}{x^3(x-1)(x-2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{x-1} + \frac{E}{x-2}$$

$$\frac{x^5-7x^3}{x^2(x-1)^2} = \underbrace{ax+b}_{\substack{\text{These are}\\ \text{calculated}\\ \text{w/ synthetic}\\ \text{Division}}} + \frac{c}{x} + \frac{d}{x^2} + \frac{e}{(x-1)} + \frac{f}{(x-1)^2}$$

$$\frac{x^5-7x^3}{x^2(x^2-2x+1)} = \frac{x^3-7x}{(x-1)^2} = \underbrace{x+2-\frac{4x+2}{(x-1)^2}}$$

This IS partial fraction rep. (basically

$$\begin{array}{r} x+2 \\ x^2-2x+1 \enclose{longdiv}{x^3+0x^2-7x} \\ \underline{x^3-2x^2+x} \\ 2x^2-8x \\ \underline{2x^2-4x+2} \\ -4x-2 \end{array}$$

$$= x+2-\frac{(4x-4)+6}{(x-1)^2}$$

$$= x+2-\frac{4}{(x-1)}-\frac{6}{(x-1)^2}$$

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$$\frac{x^3-1}{x^3-3x^2+2x}$$

$$\begin{array}{r} 1 \\ x^3-3x^2+2x \enclose{longdiv}{x^3+0x^2+0x-1} \\ \underline{x^3-3x^2+2x} \\ 3x^2-2x-1 \end{array}$$

$$\frac{x^3-1}{x(x^2-3x+2)} = 1 + \frac{3x^2-2x-1}{x(x-1)(x-2)}$$

NOTE: Numerator vanishes @ $x=1$ $\quad 3x^2-2x-1=(x-1)(3x+1)$

$$\frac{x^3-1}{x(x^2-3x+2)} = 1 + \frac{3x+1}{x(x-2)} = 1 + \frac{A}{x} + \frac{B}{x-2}$$