

### Lecture 6 Practice Exercises: Improper Integrals

At what points do the following integrals become improper?

$$\int_0^1 \frac{dx}{\sqrt{1-x^2}}$$
$$\int_0^\infty x^3 e^{-x} dx$$
$$\int_{-1}^\infty \frac{dx}{\sqrt{|x|(x^3+x-2)}}$$
$$\int_0^{\frac{\pi}{2}} \frac{\sin(x)dx}{\sqrt{\cos(x)}}$$
$$\int_0^5 x e^{-x} dx$$

Decide whether each of the above integrals converges or diverges.

Show that *for  $x$  large enough*

$$x^3 e^{-2x} \leq e^{-x}$$

(Give an estimate at how large  $x$  must be for the above inequality to hold.)

Show that

$$\int_0^\infty x^3 e^{-2x} dx$$

converges using the comparison test.