

## Math 231 Practice Exam 2

**Instructions:** This is a practice exam. Please treat it as a regular exam: sit down and take it in 50 minutes without interruption and without reference to the textbook or to the class notes. When answering questions on the convergence or divergence of a sequence or series you **MUST** give a proof or cite an appropriate theorem or theorems.

**Problem 1:**

State as briefly and concisely as you can the following theorems/tests. List all hypothesis and the conclusions. Your statement should be formulated “If (hypothesis) Then (conclusions).” If it is possible for the test to fail so indicate.

(i) **Integral Test**

(ii) **Limit Comparison Test**

(iii) **Comparison Test for SEQUENCES**

(iv)  **$k^{th}$  Term Test**

**Problem 2: Short Answer:**

(i) A sequence is decreasing ( $a_{n+1} < a_n$ ) and bounded above ( $a_n < 1$  for all  $n$ ). Is it necessarily true that the sequence converges? **Either** show that the sequence must converge **or** give an example which does not converge.

(ii) A sequence is increasing ( $a_{n+1} < a_n$ ) and bounded above ( $a_n < 1$  for all  $n$ ). Is it necessarily true that the sequence converges? **Either** show that the sequence must converge **or** give an example which does not converge.

(iii) Define what it means for a sequence to converge to a limit  $L$ .

(iv) Can a sequence  $a_n$  converge to two different limits? Either give an example of a sequence converging to two different limits **or** give a convincing argument why this is not possible.

**Problem 3:** Assess the convergence or divergence of the following sequence.  
If it converges evaluate the limit

$$a_n = \frac{n}{n+1} + (-1)^n \frac{n^2 - 1}{n^2 + 1}$$

**Problem 4:** Assess the convergence or divergence of the following series

$$\sum_{n=1}^{\infty} \frac{x-1}{\sqrt{x^{4.1}+1}}$$

**Problem 5:**

Assess the convergence or divergence of the series

$$\sum_{n=3}^{\infty} \frac{1}{(n-1)\log(n)}$$

**Problem 6:** Assess the convergence or divergence of the series

$$\sum_{n=3}^{\infty} \frac{1}{n^{2008} + 5001 \log(n) + \cos^2(n)}$$

**Problem 7:** Assess the convergence or divergence of the sequence

$$a_n = \frac{n^{2008}}{e^n + n^{2008}}$$

If the sequence converges evaluate the limit.

**Problem 8:** Does the following series converge? If so evaluate the sum

$$\sum \frac{1}{k(k+3)}$$