Math 231 Practice Exam 3

Instructions: This is a practice exam. Please treat it as a regular exam: sit down and take it in 50 minutes without interruption and without reference to the textbook or to the class notes. After this you may want to spend some time going through it carefully to see how you did. The TAs will hand out solutions in sections on Tuesday. When answering questions on the convergence or divergence of a sequence or series you **MUST** give a proof or cite an appropriate theorem or theorems.

Problem 1:

State as briefly and concisely as you can the following theorems/tests. List all hypothesis and the conclusions. Your statement should be formulated "If (hypothesis) Then (conclusions)." If it is possible for the test to fail so indicate.

(i)**Ratio Test**

(ii) **Taylor Series Formula**

(iii) **Root Test**

Problem 2: Compute the radii of convergence of the following power series:
(i)

$$\sum_{k=1}^{\infty} \frac{k^k}{k!} x^k$$

(ii)

$$\sum_{k=1}^{\infty} \frac{k^2}{k^3 + 2^k} x^k$$

(iii)

$$\sum_{k=1}^{\infty} \frac{k^2}{k!} x^k$$

Problem 3:

Compute the Taylor series for the given function about the given point: Give at least three terms. If you can guess the general form.

(i) $f(x) = \sin(x) \quad c = 1$

(ii) $f(x) = \tan(x) \quad c = 0$
Hint: You probably won't be able to guess the general form for this one.

(iii) $f(x) = e^{-x^2} \quad c = 0$

Problem 4:

All of the following series have radius of convergence equal to 1. Decide whether the series converge on the boundaries $|x - c| = \pm 1$

(i)

$$\sum_{k=0}^{\infty} \frac{x^k}{2k+1}$$

(ii)

$$\sum_{k=0}^{\infty} \frac{(x-5)^k}{k^3+11}$$

Problem 5:
Evaluate the following limits using Taylor series: (i)

$$\lim_{x \to 0} \frac{e^{x^2} - 1 - x^2}{x^3}$$

(ii)

$$\lim_{x \to 0} \frac{\sin(x^2) - x^2}{x^4}$$

(iii)

$$\lim_{x \to 0} \frac{\cos(2x) - 1 - 2x^2}{x^6}$$