Math 231 Practice Final

Instructions: This is a practice exam. Please treat it as a regular exam: sit down and take it in 3 hours without interruption and without reference to the textbook or to the class notes. After this you may want to spend some time going through it carefully to see how you did. The TAs will hand out solutions in sections on Tuesday. When answering questions on the convergence or divergence of a sequence or series you **MUST** give a proof or cite an appropriate theorem or theorems.

Problem 1:
Evaluate the following indefinite integrals

(i)

$$\int \frac{x^3}{\sqrt{1-x^2}} dx$$

(ii)

$$\int \sin^2(x) e^{3x} dx$$

(iii)

$$\int \frac{1}{x^3 - 1} dx$$

(iv)

$$\int x \arctan(x) dx$$

(v)

$$\int \frac{1}{x^2(x-1)^2} dx$$

(iv)

$$\int \sec(x) \tan^2(x) dx$$

(vi)

$$\int \cos^2(x) \sin(2x) dx$$

Problem 2: Compute the radii of convergence of the following power series. If the series has a finite radius of convergence check the endpoints to determine if the series converges there.

(i)

$$\sum_{k=1}^{\infty} \frac{k!}{k^{2k}} x^k$$

(ii)

$$\sum_{k=1}^{\infty} \frac{k^2}{k^3 + \sin(k)} x^{2k}$$

(iii)

$$\sum_{k=1}^{\infty} e^{-k \ln(k)} x^k$$

Problem 3:
Consider the following parametric curve:

$$(x(t), y(t)) = (3\cos(2t) - 2\sin(3t), 3\sin(2t) + 2Cos(3t))$$

which looks like the following:

(i) Compute the tangent (velocity) vector to the curve.

(ii) Compute the length of the velocity vector **HINT:** The identity $\cos(a)\sin(b) - \sin(a)\cos(b) = \sin(b-a)$ may be useful.

(iii) Where does the tangent vector have zero length?

(iv) Compute the integral

$$\int_0^{2\pi} y\frac{dx}{dt}dt.$$

What area does this represent?

(v) Compute the arclength of the curve over $t \in (0, 2\pi)$

Problem 4:
Evaluate the convergence or divergence of the following series
(i)

$$\sum_{k=2}^{\infty} \frac{1}{k + k^{1/2}\sin(k)}$$

(ii)

$$\sum_{k=0}^{\infty} \frac{1}{k + k^{3/2}\sin(k)}$$

(iii)

$$\sum_{k=0}^{\infty} \frac{(-1)^k}{k + k^{1/2}}$$

(iii)

Problem 5:

Evaluate the following limits using Taylor series: (i)

$$\lim_{x \to 0} \frac{e^{x^2} - 1 - x^2}{x^2}$$

(ii)

$$\lim_{x \to 0} \frac{\sin(x^2) - x^2}{x^6}$$

(iii)

$$\lim_{x \to 0} \frac{\cos(2x) - 1 - 2x^2}{x^6}$$

Problem 6:
Find the Taylor series for the following functions about the indicated point
(i) $f(x) = \sin(x) \qquad c = \frac{\pi}{4}$

(ii) $f(x) = \ln(\frac{1+x}{1-x}) \qquad c = 0$ **HINT:** Recall that $\ln(a/b) = \ln(a) - \ln(b)$

(iii) $f(x) = \ln(\frac{1+x}{1-x}) \qquad c = 0$ **Hint: This is tricky! What is** $\frac{d^4 f}{dx^4}$

Problem 7:
Consider the following parametric curve:

$$(x(t), y(t)) = (t - \sin(3t), 1 - 3\cos(3t))$$

which looks like the following:

(i) Compute the tangent (velocity) vector to the curve.

(ii) Compute the length of the velocity vector **HINT:** The identity $\cos(a)\sin(b) - \sin(a)\cos(b) = \sin(b - a)$ may be useful.

(iii) Where does the tangent vector have zero length?

(iv) Compute the integral

$$\int_0^{2\pi} y\frac{dx}{dt}dt.$$

What area does this represent?

(v) Compute the arclength of the curve over $t \in (0, 2\pi)$