Math 341 Computer Lab - December 6, 2002

In today's lab we will use the phase plane plotting feature of IODE - option [2] of the main menu - to explore second order nonlinear equations.

Reminders:

To start IODE type **cd IODE** at the unix prompt, to change directory to the IODE main directory. Then type **octave** to start octave running, and finally type **iode** at the octave prompt to load the IODE package.

If you have any trouble you will find the login instructions from the first lab are attached at the end of the lab instructions.

When entering equations remember that IODE uses * for multiplication. An expression like 4y will generate an error: 4*y is the correct way to enter this quantity.

0.1 Starting Simple:

Exercise 0:

Given the linear system

$$\begin{aligned}\frac{dy}{dt} &= ax + by \\ \frac{dx}{dt} &= cx + dy\end{aligned}$$

Find values of a, b, c, d such that the origin is a

- center (Two purely imaginary roots)
- spiral source (Two complex roots with positive real part)
- saddle (two real roots - one positive, one negative)

0.2 A simple nonlinear system

Next we consider a simple nonlinear equation:

$$\frac{d^2y}{dt^2} + y(1-y) = 0.$$

Exercise 1: Write the above second order system as a first order system in the form

$$\begin{aligned}\frac{dy}{dt} &= F(x,y) \\ \frac{dx}{dt} &= G(x,y)\end{aligned}$$

Exercise 2:

Find the steady states (i.e. values of x^*, y^* such that $F(x^*, y^*) = G(x^*, y^*) = 0$. Also calulate the nature of the steady state in each case (sink, source, center, spiral sink, saddle, etc..)

Hint: There should be two fixed points.

Exercise 3: Use option [2] from the IODE main menu to plot the phase plane for the above system. Find the steady states on your plot. Does picture agree with what you calculated for the nature of the steady state?

0.3 A slightly more complicated system:

The Lotka-Volterra equations of mathematical biology are given by

$$\begin{aligned}\frac{dx}{dt} &= x - xy \\ \frac{dy}{dt} &= -y + xy\end{aligned}$$

x, y are supposed to represent the populations of a prey species and a predator species (rabbits and foxes, for instance). The frequency at which rabbits are eaten by foxes is taken to be proportional to xy. So the first equation says that rate of change of the number of rabbits is proportional to the current population minus the number eaten by foxes. The second says that the rate of change of the number of foxes is equal to the number of foxes who catch foxes minus the total fox population. (The idea is that the foxes who are able to catch rabbits can reproduce, and the one who don't tend to starve.)

Note that if there are no foxes $y = 0$ then x, number of rabbits, grows exponentially. If there are no rabbits $x = 0$ the number of foxes decays exponentially.

Exercise 4: Find all of the steady states. Determine the nature of these fixed points (center, node, saddle, etc)

Exercise 5: Plot the phase plane and a number of numerical solutions to the system using the phase plane plotter in IODE.

Excercise 6: What does this model predict about the rabbit population? Does it stay fixed? Explain this in terms of the rabbit/fox model.

0.4 Centers and periodic orbits

It was stated in class that, even if the linearized equation has a center, the full equation need not. We explore this further in this problem.

Consider the system

$$\begin{aligned}\frac{dx}{dt} &= y + x(x^2 + y^2) \\ \frac{dy}{dt} &= -x + y(x^2 + y^2)\end{aligned}$$

Exercise 7: Show that $(0,0)$ is the only fixed point. Show that the linearization about this point is a center.

Exercise 8: Plot the solution to the above. Are the solution curves circles? Explain.

Exercise 9: Let $\rho = x^2 + y^2$. If the solution curves are circles, then ρ should be constant. Compute $d\rho/dt$ using the above equation? Is it zero?

Exercise 10: Solve the equation for rho you derived in the previous part. You should find that ρ grow monotonically with t.

Appendix

Logging In: The login name for the class account is "math341" (all lower case) and the password is *****. Please note that UNIX is *case sensitive.*

If you have a math department account please use that, rather than the class account.

Getting a Window

After you have successfully logged in you will need to open a terminal window. If there is no terminal window open you will need to open one. To do so click on the background with the right mouse button. You should get a menu titled "Workspace Menu". Go to the "Tools" submenu and select "Terminal". This should give you a terminal window.

Finding the IODE directory:

After you have gotten a terminal menu you will need to move to the IODE directory. You can do this by typing "cd iode" at the unix prompt, as follows:

```
u57 1% cd IODE
```

which will **c**hange **d**irectory to the IODE directory.

Note: The aboves assumes that you are sitting at the machine named u57 - your prompt will be slightly different.

Starting Octave/IODE:

The IODE package runs under Octave (a free software package which is largely compatible with Matlab). To start Octave type octave at the unix prompt:

```
u57 2% octave
```

From the Octave prompt simply type "iode" to bring up the iode package"

```
GNU Octave, version 2.0.16 (sparc-sun-solaris2.7).
Copyright (C) 1996, 1997, 1998, 1999, 2000 John W. Eaton.
This is free software with ABSOLUTELY NO WARRANTY.
For details, type ‘warranty’.

octave:1> iode
```

You should get a prompt which closely resembles the following

```
$Id: iode.m,v 1.21 2002-06-03 19:57:31-05 brinkman Exp $
Copyright (c) 2001, Peter Brinkmann (brinkman@math.uiuc.edu)
This program is distributed in the hope that it will be useful,
but WITHOUT ANY WARRANTY; without even the implied warranty of
MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE.  See the
GNU General Public License for more details.

Running under Octave.


Iode Main Menu

  [ 1] Direction fields
  [ 2] Phase planes
  [ 3] Second order linear ODEs
  [ 4] Fourier series
  [ 5] Partial differential equations
  [ 6] Quit

pick a number, any number:
```