Math 213 - Extra Problems

These problems will be assigned regularly throughout the semester. If a student has the grade of A at the end of the semester and has successfully completed 10 points worth of these problems, they will be awarded the grade of A+. The number of points assigned to a problem corresponds to its difficulty level.

These problems may be turned in up to and including the end of the last class period.

(1) Problem 2.2.40 from the book (1 point).
(2) Problem 2.4.48 from the book (1 point).
(3) Let A and B be sets. Suppose that $f : A \to B$ and $g : B \to A$ are one-to-one. Show that there is a bijection from A to B. (7 points).
(4) Show that $\ln(n!) = n\ln(n) - n + O(\ln(n))$ (you may not assume the result stated on pg. 146). (1 point).
(5) Problem 4.1.47 from the book (please don't look up the solution). (1 point).
(6) Prove that for all $n \geq 1$,

$$\int_0^\infty x^n e^{-x}\, dx = n!.$$

(1 point).
(7) Prove that for all $n \geq 0$, if F_n is the nth Fibonacci number, then

$$F_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right].$$

(1 point).
(8) How many ways are there to get a full house poker hand, drawn from a deck of 52 cards? [A full house is a collection of 5 cards where two of the cards have the same value (2, 3, 4, 5, ..., 10, J, Q, K, or A), and the other three also have the same value]. (1 point).
(9) 5.2.26 (2 points). [This problem uses 5.2.25. Reading the solution to 5.2.25 might help know how to start.]
(10) 5.3.42 (2 points).
(11) 5.4.32 (2 points).
(12) How many ways are there to give 1,000,000 dollars to the 45 students in both sections so that the number of dollars each student receives is a positive integer? (2 points).
(13) Suppose that a fair coin is flipped continually until it lands heads. Let X be the random variable giving the number of times the coint was flipped. Show that $E(X) = 2$. (1 point).
(14) Suppose that a coin is flipped n times and lands heads with probability p. Let X be the random variable that is the number of times it landed heads. Show that $V(X) = np(1-p)$. (2 points).

(15) Suppose that a large calculus lecture takes an exam. Show that on this exam, at least 75 percent of the students were between two standard deviations below the mean, and two standard deviations above the mean. (1 point).
(16) 7.1.34. (1 point).
(17) Suppose $a_1 = 4$, $a_2 = 11$, $a_3 = 33$ and for $n \geq 4$,

$$a_n = (n+6)a_{n-1} - (6n+3)a_{n-2} + (9n-18)a_{n-3}.$$

Prove by strong induction that

$$a_n = n! + 3^n$$

for all $n \geq 1$. (1 point).
(18) Suppose that $A_1, A_2, \ldots, A_9$ are sets each with size 4. They all have one element in common, and if i and j are distinct, then $|A_i \cap A_j| = 1$ or 2. What is the largest and smallest possible sizes for their union? (3 points).
(19) 6.2.39. (4 points).
(20) How many relations on a set with n elements are symmetric? (1 point).
(21) How many relations on a set with n elements are antisymmetric? (1 point).
(22) How many relations on a set with n elements are both symmetric and antisymmetric? (1 point).
(23) 9.3.44. (1 point).
(24) Prove Theorem 1 from Section 9.2 by counting the number of 1s in an adjacency matrix in two different ways. (1 point).
(25) Show that the graph pictured in exercise 9.5.46 is isomorphic to the graph pictured on the course website. (1 point).
(26) Show that if G is a connected graph with exactly two vertices a and b of odd degree, and if $\deg(a) > 1$, then one of the edges incident to a is not a cut edge. (1 point).
(27) If $n \geq 1$, find a planar representation of $K_{n,2}$. (1 point).
(28) Show that Q_4 is nonplanar. (1 point).
(29) 9.7.36. (1 point).
(30) 9.8.36. (2 points).