Math 213 - Homework 4

Assigned: 9/12/07

Due: 9/19/07 at the start of class.

Notation: Exercise a.b.c(d) stands for part (d) of Exercise c from Section a.b.

Problems:

(1) 4.2.2.
(2) 4.2.6(a) and (c).
(3) Show that if n is a positive integer, then $n - \lfloor \frac{n}{2} \rfloor = \lfloor \frac{n+1}{2} \rfloor$. [You don't need induction here. Consider the cases when n is even and n is odd separately.]
(4) Let $g(n)$ denote the number of comparisons necessary to merge sort a list of length n. Show that $g(1) = 0$, $g(2) = 1$, and $g(3) = 3$.
(5) Explain why
$$g(n) = g\left(\left\lfloor \frac{n}{2} \right\rfloor\right) + g\left(\left\lfloor \frac{n+1}{2} \right\rfloor\right) + (n-1).$$
(6) Use strong induction to prove that for all $n \geq 1$, $g(n) \leq g(n+1)$. [The induction hypothesis here is that $g(k) \leq g(k+1)$ for all $k \leq n$.]
(7) Prove by regular induction that $g(2^n) = (n-1)2^n + 1$.
(8) Show that $g(n) = O(n \log(n))$. [For any n, there is some l with $n \leq 2^l \leq 2n$. Then, by problem 6, $g(n) \leq g(2^l)$. Use problem 7 and that $l \leq \log(2n)$].
(9) Show that if $n \geq 16$, then $n \leq \frac{n}{4} \log(n)$.
(10) Show that $g(n) = \Omega(n \log(n))$. [For any n, there is some l with $n/2 \leq 2^{l-1} \leq n$. Note that $l \geq \log(n)$].