Math 213 - Homework 8

Assigned: 10/10/07

Due: 10/17/07 at the start of class.

Notation: Exercise a.b.c(d) stands for part (d) of Exercise c from Section a.b.

Problems:

(1) 7.1.2(e).
(2) 7.1.14. [Part (c) is similar to solving the recurrence from example 5].
(3) 7.1.24(a).
(4) 7.1.28(a).
(5) Define $w_0 = 1$, and for $n \geq 1$, $w_n = w_1 \cdot w_2 \cdots w_{n-1} + 1$. (That is, w_n is one more than the product of the previous terms). Show that $w_n = w_{n-1}^2 - w_{n-1} + 1$ for $n \geq 1$.
(6) 7.2.6.
(7) 7.2.8.
(8) 7.2.18.
(9) 7.2.22.
(10) A frog jumps between two lillypads. It starts out on lillypad 1 at midnight. If the frog is on lillypad 1 n minutes after midnight, there is a 50 percent chance it jump to lillypad 2 and stay there for a minute, and a 50 percent chance it will stay on lillypad 1 for a minute. If the frog is on lillypad 2 n minutes after midnight, it will jump back to lillypad 1 for a minute. Let a_n be the probability that the frog is on lillypad 1 at the end of n minutes (so $a_0 = 1$ and $a_1 = 1/2$). Show that for $n \geq 2$,

$$a_n = 1 - \frac{1}{2} a_{n-1}.$$