

When can a triple integral be reduced to a line integral? More precisely, we take a function $f : \mathbb{R}^3 \rightarrow \mathbb{R}$ and integrate it over a region W , namely $\iiint_W f \, dV$. Apply the Divergence theorem giving $\iiint_W f \, dV = \iint_{\partial W} \vec{G} \cdot d\vec{S}$, where $\nabla \cdot \vec{G} = f$. Then, apply Stokes theorem and get $\iint_{\partial W} \vec{G} \cdot d\vec{S} = \int_{\partial(\partial W)} \vec{H} \cdot d\vec{s}$, where $\nabla \times \vec{H} = \vec{G}$. In particular, what does $\partial(\partial W)$ look like? What properties must f have in order to be able to find vector fields \vec{G} and \vec{H} ?