

Math 241 C1H - Problems to Think About - March 31, 2008

Instructions: Pair up with two other students and work on the problems below in your group. If you think you have the answer to a problem, write down your answer and your reasoning.

1. Let $D^* = [0, 1] \times [0, 1]$, let $T : D^* \rightarrow \mathbb{R}^2$ be given by $T(u, v) = (x(u, v), y(u, v)) = (3u + 4v, 2u + 3v)$.

(a) What does $T(D^*)$ look like? What is its area? [Do this using geometry alone].

(b) Compute $\frac{\partial(x,y)}{\partial(u,v)}$. Use this to recompute the area of $T(D^*)$ and check your answer from (a).

2. Let C be the region inside a cylinder of radius r_0 and height h , and let D be the region inside a donut with minor radius r_0 and major radius $h/2\pi$ (so the circle going through the middle of the donut has the same length as the cylinder).

(a) What is the volume of C ?

(b) Imagine a transformation $T : C \rightarrow D$ that bends the cylinder into the shape of the donut by attaching the top and the bottom of the cylinder together. Let $|\frac{\partial(x,y)}{\partial(u,v)}|$ be the Jacobian of this transformation. Where is the Jacobian greater than 1? Where is it less than 1? Where is it equal to 1?

(c) Is the volume of D greater or less than the volume of C ? [I once lost a bet about the answer to this question.]

(d) Check your answer to (c) by computing the volume of D . [Bounds for D given in cylindrical coordinates are $-r_0 \leq z \leq r_0$, $0 \leq \theta \leq 2\pi$, and $h/2\pi - \sqrt{r_0^2 - z^2} \leq r \leq h/2\pi + \sqrt{r_0^2 - z^2}$.]

3. Let S_1 be the sphere $x^2 + y^2 + z^2 = 1$, and let S_2 be the sphere $x^2 + y^2 + (z - 1)^2 = 1$. Compute the volume of the region that is inside both S_1 and S_2 .