Math 241 - Section C1H - Homework 14

Assigned: 4/24/08
Due: 4/30/08 at the start of class.
Notation: Exercise a.b.c stands for Exercise c from Section a.b.
Problems to hand in:

(1) 8.2.2.
(2) 8.2.6.
(3) 8.2.22.
(4) 8.2.29.
(5) Compute the surface integral $\iint_S \vec{F} \cdot d\vec{S}$, where $\vec{F}(x, y, z) = (x, 0, 0)$ and S consists of the six faces of the cube $\{(x, y, z) : -1 \leq x, y, z, \leq 1\}$. What does the Divergence theorem tell you the answer should be?
(6) 8.3.2.
(7) 8.3.6.
(8) 8.3.16.
(9) 8.3.18.
(10) For each of the examples 7.1, 7.2, 7.3, and 7.4 (from Section 7.1) determine the boundary of the surfaces described, and whether or not the parametrizations $r(u, v)$ are one-to-one.
(11) 8.3.24.
(12) When can a triple integral be reduced to a line integral? More precisely, we take a function $f : \mathbb{R}^3 \to \mathbb{R}$ and integrate it over a region W, namely $\iiint_W f\, dV$. Apply the Divergence theorem giving $\iiint_W f\, dV = \iint_{\partial W} \vec{G} \cdot d\vec{S}$, where $\nabla \cdot \vec{G} = f$. Then, apply Stokes theorem and get $\iint_{\partial W} \vec{G} \cdot d\vec{S} = \int_{\partial(\partial W)} \vec{H} \cdot d\vec{s}$, where $\nabla \times \vec{H} = \vec{G}$. In particular, what does $\partial(\partial W)$ look like? What properties must f have in order to be able to find vector fields $\vec{G}$ and $\vec{H}$?
(13) Let $\vec{F}(x, y, z) = (y, z, x)$. Find a vector field $\vec{G}$ with $\nabla \times \vec{G}(x, y, z) = \vec{F}$.
(14) Use Stokes' theorem and the result of the previous problem to compute $\iint_S \vec{F} \cdot d\vec{S}$ over any surface whose boundary is the unit circle $x^2 + y^2 = 1$ oriented counterclockwise.