Math 595 - Using MAGMA to compute with modular forms

The second problem on the homework assignment is not feasible without some help from a computer. This file is to illustrate some of the features that MAGMA has.

First, to access MAGMA, log into any Linux computer in the math department, open a terminal, type `magma` and press enter.

To work with a space of modular forms one can enter commands as follows:

```
M := ModularForms(4,2);
```

This creates the space of modular forms of level 4 and weight 2. One of the most useful commands is

```
Basis(M);
```

To increase the precision enter

```
SetPrecision(M,1000);
```

To create modular forms with non-trivial character one can do the following. First, create the character:

```
chi := DirichletGroup(16)!KroneckerCharacter(-4);
```

This means create the character χ defined by $\chi(d) = \left(\frac{-4}{d}\right)$, but make it a character mod 16. One can create a space of modular forms with character as follows.

```
M := ModularForms(chi,3);
```

This means the space $M_3(\Gamma_0(16), \chi)$. Here are some other useful commans.

```
C := CuspidalSubspace(M);
```

```
SetPrecision(C,100);
```

```
Basis(C);
```

To work with expressions involving the Dedekind η-function, one can do the following:

```
R<q> := PuiseuxSeriesRing(RationalField() :  Precision := 1000);
```

A Puiseux series ring is a type of power series ring where fractional exponents are allowed.

```
f := DedekindEta(q^4)^6;
```

```
f;
```

The form f is the q-expansion of $\eta^6(4z)$. The form f should be the same as the basis element of C found above.

MAGMA also has routines for working with Eisenstein series and Delta.

```
Eisenstein(4,q)^2 - Eisenstein(8,q);
```

```
Delta(q)*Eisenstein(4,q);
```

One can also work with Hecke operators in MAGMA.

```
C2 := CuspForms(37,2);

SetPrecision(C2,200);

R<x> := PolynomialRing(RationalField());

T13 := HeckeOperator(C2,13);

T1 := HeckeOperator(C2,1);

f := CharacteristicPolynomial(T13);

Factorization(f);

Kernel(T13+4*T1);

Kernel(T13+2*T1);

B := Basis(C2);

f1 := B[1];

f2 := B[1] - 2*B[2];

f1*T13 + 4*f1;

f2*T13 + 2*f2;
```

For MAGMA's documentation, which includes more examples, go to `http://magma.maths.usyd.edu.au/magma/htmlhelp/MAGMA.htm` (this is the documentation for version 2.15, the department has an older version 2.12, so some new features may not work). From here, click on "Modular arithmetic geometry," then "Modular forms."

When you're done using MAGMA, exit by typing

```
quit;
```