

Math 597 - Homework 2

1. (a) Let $f(z) = \eta(z)^8 \eta(2z)^8$. Show that the map $\Phi : M_k(\Gamma_0(2)) \rightarrow S_{k+8}(\Gamma_0(2))$ given by $\phi(g) = fg$ is an isomorphism.

(b) Use the fact that $\dim M_2(\Gamma_0(2)) = 1$ to find a formula for $\dim M_k(\Gamma_0(2))$ for all k .

2. Let

$$f(z) = \frac{\eta(8z)^8}{\eta(4z)^2 \eta(16z)^2} = \sum_{n=1}^{\infty} a(n)q^n.$$

(a) Show that $f(z) \in S_2(\Gamma_0(64))$. [You may wish to look up Ken's book. It gives a formula for the order of vanishing of an eta-quotient at each cusp].

(b) Show that if $\gcd(m, n) = 1$ then $a(mn) = a(m)a(n)$, and

$$a(p^n) = a(p)a(p^{n-1}) - pa(p^{n-2}) \text{ if } n \geq 2, p \neq 2,$$

and $a(2^n) = 0$.