

Math241, Quiz 3-version a, Oct 1

Name: Solutions.

Question 1: [5pt] Use implicit differentiation to find $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$.

$$xz = \ln(y^2 + z)$$

Sol) For $\frac{\partial z}{\partial x}$, $z + x \frac{\partial z}{\partial x} = \frac{1}{y^2 + z} \frac{\partial z}{\partial x}$.
product rule chain rule.

then solve for $\frac{\partial z}{\partial x}$, $z = \left(\frac{1}{y^2 + z} - x \right) \frac{\partial z}{\partial x}$.

$$\frac{\partial z}{\partial x} = \frac{z}{\frac{1}{y^2 + z} - x} = \frac{zy^2 + z^2}{1 - xy^2 - xz},$$

Similarly for $\frac{\partial z}{\partial y}$,

$$x \frac{\partial z}{\partial y} = \frac{1}{y^2 + z} \left(2y + \frac{\partial z}{\partial y} \right),$$

$$\left(x - \frac{1}{y^2 + z} \right) \frac{\partial z}{\partial y} = \frac{2y}{y^2 + z},$$

$$\frac{\partial z}{\partial y} = \frac{\frac{2y}{y^2 + z}}{x - \frac{1}{y^2 + z}} = \frac{2y}{xy^2 + xz - 1}.$$

Question 2: [5pt] Find an equation of the tangent plane to the following surface at the given point.

$$z = y \ln(\sqrt{x}), \quad (1, 4, 0)$$

Sol) Let $f(x, y) = y \ln(\sqrt{x}) = \frac{1}{2} y \ln x$, $x > 0$.

$$z = f(1, 4) + f_x(1, 4)(x-1) + f_y(1, 4)(y-4) \neq \emptyset,$$

$$f(1, 4) = 0.$$

$$f_x = \frac{1}{2} y \cdot \frac{1}{x} = \frac{y}{2x}, \quad f_x(1, 4) = \frac{4}{2} = 2.$$

$$f_y = \frac{1}{2} \ln x, \quad f_y(1, 4) = 0,$$

$$\text{Hence } z = 0 + 2(x-1) + 0(y-4) \neq \emptyset,$$

$$\cancel{x-1=0},$$

$$z = 2(x-1)$$