Math241, Quiz 3-version b, Oct 1

Name: Solutions.

Question 1: [5pt] Use implicit differentiation to find $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}$.

$$xyz = \ln(x+y+z)$$

Sol) For $\frac{\partial z}{\partial x}$, $\underbrace{yz + xy\frac{\partial z}{\partial x}}_{\text{product rule}} = \underbrace{\frac{1}{x+y+z}\left(1+\frac{\partial z}{\partial x}\right)}_{\text{chain rule}}$.

Solve for $\frac{\partial z}{\partial x}$.

$$\left(xy - \frac{1}{x+y+z}\right)\frac{\partial z}{\partial x} = \frac{1}{x+y+z} - yz$$

So $$\frac{\partial z}{\partial x} = \frac{\frac{1}{x+y+z} - yz}{xy - \frac{1}{x+y+z}} = \frac{1 - xyz - y^2z - yz^2}{x^2y + xy^2 + xyz - 1},$$

Similarly, for $\frac{\partial z}{\partial y}$,

$$xz + xy\frac{\partial z}{\partial y} = \frac{1}{x+y+z}\left(1+\frac{\partial z}{\partial y}\right).$$

$$\frac{\partial z}{\partial y} = \frac{1 - xyz - x^2z - xz^2}{x^2y + xy^2 + xyz - 1}.$$

Question 2: [5pt] Find an equation of the tangent plane to the following surface at the given point.

$$z = y^2 \ln(x^2), \quad (-1, 2, 0)$$

Sol) Let $f(x,y) = y^2 \ln(x^2)$.

$Z = f(-1,2) + f_x(-1,2)(x+1) + f_y(-1,2)(y-2) \cancel{=0}$

$f(-1,2) = 0$,

$f_x = y^2 \cdot \dfrac{2x}{x^2} = \dfrac{2y^2}{x}$, $f_x(-1,2) = \dfrac{8}{-1} = -8$,

$f_y = 2y \ln(x^2)$, $f_y(-1,2) = 0$,

$Z = 0 - 8(x+1) + 0 \cdot (y-2) \cancel{=0}$,

Hence ~~$x+1=0$.~~ $Z = -8(x+1)$