Math241, Quiz 4-version a, Oct 8

Name: Solutions.

Question 1: [5pt] Find $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}$.

$$z^3 = xy^2 + x^2 z$$

Sol) Let $F(x, y, z) = z^3 - xy^2 - x^2 z$.

Then $\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{-y^2 - 2xz}{3z^2 - x^2} = \frac{y^2 + 2xz}{3z^2 - x^2}$,

$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{-2xy}{3z^2 - x^2} = \frac{2xy}{3z^2 - x^2}$.

Question 2: [5pt] Find the local maximum and minimum values and saddle points of the function.

$$f(x, y) = x^4 + 36y^3 - 12xy + 1$$

Sol) $f_x = 4x^3 - 12y = 0$, $x^3 - 3y = 0$ $\cdots$ (*)

$f_y = 3 \cdot 36y^2 - 12x = 0$, $9y^2 - x = 0$:

From (*), $y = \frac{x^3}{3}$.

Plug in the second equation.

$9 \cdot \left(\frac{x^3}{3}\right)^2 - x = 0$, $x^6 - x = 0$, $x(x^5 - 1) = 0$

$x = 0, 1$.

Critical points are $(0,0), (1, \frac{1}{3})$.

For $D = f_{xx} f_{yy} - f_{xy}^2$,

$f_{xx} = 12x^2$, $f_{xy} = -12$, $f_{yy} = 6 \cdot 36y$.

$D = 12x^2 \cdot 6 \cdot 36 \cdot y - 12^2$

$D(0,0) = -12^2 < 0$ saddle point

$D(1, \frac{1}{3}) = 12 \cdot 6 \cdot \cancel{36}^{12} \cdot \frac{1}{\cancel{3}} - 12^2 > 0$.

But $f_{xx}(1, \frac{1}{3}) = 12 > 0$. local minimum.

$f(1, \frac{1}{3}) = 1 + 36\left(\frac{1}{3}\right)^3 - 12 \cdot \frac{1}{3} + 1$

$= 1 + \frac{\cancel{36}\,4}{\cancel{27}\,3} - 4 + 1$

$= \frac{4}{3} - 2 = -\frac{2}{3}$. min. value.