Math241, Quiz 4-version b, Oct 8

Name: SOLUTIONS

Question 1: **[5pt]** Find $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}$.

$$xy^2 = z^3 + z^2x$$

Sol) Let $F(x,y,z) = xy^2 - z^3 - z^2x$.

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{y^2 - z^2}{-3z^2 - 2zx} = \frac{y^2 - z^2}{3z^2 + 2zx},$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{2xy}{-3z^2 - 2zx} = \frac{2xy}{3z^2 + 2zx}.$$

Question 2: [5pt] Find the local maximum and minimum values and saddle points of the function.

$$f(x,y) = 36x^3 + y^4 - 12xy + 14$$

Sol) For critical points,

$f_x = 3\cdot 36x^2 - 12y = 0 \Rightarrow y = 9x^2$

$f_y = 4y^3 - 12x = 0 \Rightarrow x = \frac{y^3}{3}$.

$y = 9\cdot\left(\frac{y^3}{3}\right)^2$, $y = y^6$, $y(y^5-1) = 0$. $y = 0, 1$.

$y = 0 \Rightarrow x = 0$, $y = 1 \Rightarrow x = \frac{1}{3}$.

$(0,0), \left(\frac{1}{3}, 1\right)$.

For D, $f_{xx} = 6\cdot 36x$, $f_{xy} = -12$, $f_{yy} = 12y^2$,

$D = f_{xx}f_{yy} - f_{xy}^2 = 6\cdot 36x\cdot 12y^2 - 12^2$,

$D(0,0) = -12^2 < 0$, saddle points.

$D\left(\frac{1}{3},1\right) = 6\cdot 36\cdot\frac{1}{3}\cdot 12 - 12^2 = 6\cdot 12^2 - 12^2 > 0$.

But $f_{xx}\left(\frac{1}{3},1\right) = 6\cdot 36\cdot\frac{1}{3} > 0$ local minimum.

$f\left(\frac{1}{3},1\right) = 36\cdot\frac{1}{27} + 1 - 12\cdot\frac{1}{3} + 14$

$= \frac{4}{3} + 11 = 12\frac{1}{3} = \frac{37}{3}$. : local min. value.