Math241, Quiz 5-version b, Oct 15

Name: Solutions.

Question 1: **[5pt]** Find the maximum and minimum values of the function $f(x, y, z)$ subject to the given constraints. Hint:use Lagrange multipliers.

$$f(x, y, z) = x^2 - y^2 + z^2; \quad x^4 + y^4 + z^4 = 2$$

Sol) $\nabla f = \lambda \nabla g$, $g(x,y,z) = x^4 + y^4 + z^4$.

$\langle 2x, -2y, 2z \rangle = \lambda \langle 4x^3, 4y^3, 4z^3 \rangle$.

$$\begin{cases} 2x = 4x^3\lambda, & 2x(1-2x^2\lambda) = 0, & x = 0 \text{ or } x^2 = \frac{1}{2\lambda}, \\ -2y = 4y^3\lambda, & 2y(1+2y^2\lambda) = 0, & y = 0 \text{ or } y^2 = -\frac{1}{2\lambda}, \\ 2z = 4z^3\lambda, & 2z(1-2z^2\lambda) = 0, & z = 0 \text{ or } z^2 = \frac{1}{2\lambda}. \\ x^4+y^4+z^4 = 2 \end{cases}$$

Note that $\lambda \neq 0$ if so, $x = y = z = 0$ but $(0,0,0)$ is not on the surface $x^4+y^4+z^4 = 2$.

① $x = y = 0$, $z^2 = \frac{1}{2\lambda}$, $(0,0,\pm\sqrt[4]{2})$, $f(0,0,\pm\sqrt[4]{2}) = \sqrt{2}$,

② $x = z = 0$, $y^2 = -\frac{1}{2\lambda}$, $(0, \pm\sqrt[4]{2}, 0)$, $f(0,\pm\sqrt[4]{2},0) = -\sqrt{2}$ min.val.

③ $y = z = 0$, $x^2 = \frac{1}{2\lambda}$, $(\pm\sqrt[4]{2},0,0)$, $f(\pm\sqrt[4]{2},0,0) = \sqrt{2}$,

④ $y = 0$, $x^2 = \frac{1}{2\lambda}$, $z^2 = \frac{1}{2\lambda}$, $\frac{1}{4\lambda^2} + \frac{1}{4\lambda^2} = 2$, $\lambda^2 = \frac{1}{4}$, $\lambda = \pm\frac{1}{2}$, $\lambda = \frac{1}{2}$.

$x^2 = z^2 = 1$, $(\pm 1, 0, \pm 1)$, $f(\pm 1, 0, \pm 1) = 2$ max.val,

Note that $y^2 = -\frac{1}{2\lambda}$, $z^2 = \frac{1}{2\lambda}$ does not occur at the same time.

Question 2: [5pt] Find the volume of the solid under the hyperbolic paraboloid $z = 16 - x^2 + y^2$ and above the square region $R = [1,4] \times [0,1]$.

Sol) $\displaystyle\int_1^4 \int_0^1 16 - x^2 + y^2 \, dy \, dx = \int_1^4 \left(16 - x^2 + \frac{1}{3}\right) dx$

$\displaystyle = \left[16x - \frac{x^3}{3} + \frac{1}{3}x\right]_1^4 = 64 - \frac{64}{3} + \frac{4}{3} - \left(16 - \cancel{\frac{1}{3}} + \cancel{\frac{1}{3}}\right)$

$\displaystyle = 48 - \frac{60}{3} = 48 - 20 = 28.$