

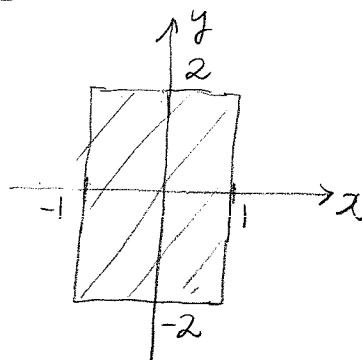
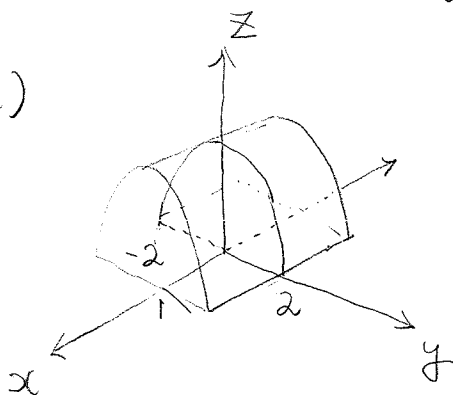
Math241, Quiz 6-version a, Oct 29

Name: Solutions.

Question 1: [5pt] Let  $E$  be a region bounded by the parabolic cylinder  $z = 4 - y^2$  and the planes  $z = 0$ ,  $x = -1$  and  $x = 1$ . Compute

$$\iiint_E x e^y dV$$

Sol.)

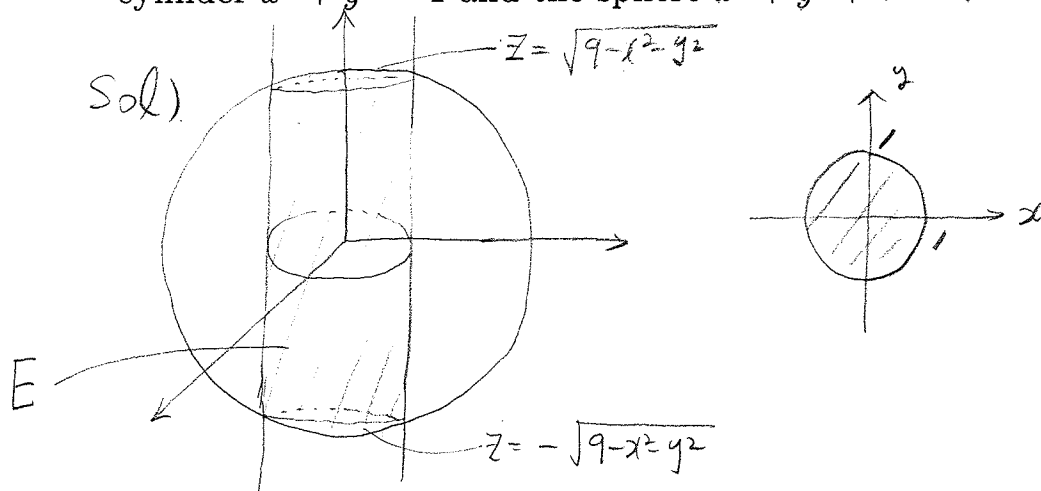


$$z = 0, \quad y^2 = 4, \quad y = \pm 2.$$

$$\int_{-1}^1 \int_{-2}^2 \int_0^{4-y^2} x e^y dz dy dx = \int_{-1}^1 \int_{-2}^2 x e^y (4 - y^2) dy dx =$$

$$\int_{-1}^1 x \int_{-2}^2 e^y (4 - y^2) dy dx = \int_{-2}^2 e^y (4 - y^2) dy \int_{-1}^1 x dx = 0.$$

**Question 2:** [5pt] Find the volume of the solid that lies within both the cylinder  $x^2 + y^2 = 1$  and the sphere  $x^2 + y^2 + z^2 = 9$ .



$$V = \iiint_E dV = \iint_D \int_{-\sqrt{9-x^2-y^2}}^{\sqrt{9-x^2-y^2}} dz dA = \int_0^{2\pi} \int_0^1 \int_{-\sqrt{9-r^2}}^{\sqrt{9-r^2}} r dz dr d\theta =$$

$$\int_0^{2\pi} \int_0^1 2r\sqrt{9-r^2} dr d\theta = 2\pi \left[ -\frac{2}{3} (9-r^2)^{\frac{3}{2}} \cdot (-1) \right]_0^1$$

$$= -\frac{4\pi}{3} \left[ (9-r^2)^{\frac{3}{2}} \right]_0^1 = -\frac{4\pi}{3} \left( 8^{\frac{3}{2}} - 9^{\frac{3}{2}} \right) = \frac{4\pi}{3} \left( 3^3 - 2^{\frac{9}{2}} \right) =$$

$$\frac{4\pi}{3} (27 - 16\sqrt{2})$$