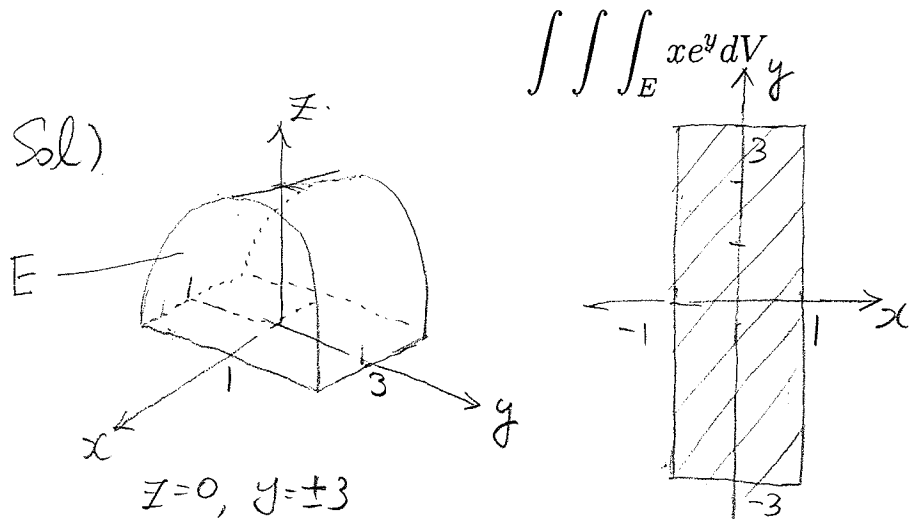


Math241, Quiz 6-version b, Oct 29

Name: Solutions

Question 1: [5pt] Let  $E$  be a region bounded by the parabolic cylinder  $z = 9 - y^2$  and the planes  $z = 0$ ,  $x = -1$  and  $x = 1$ . Compute

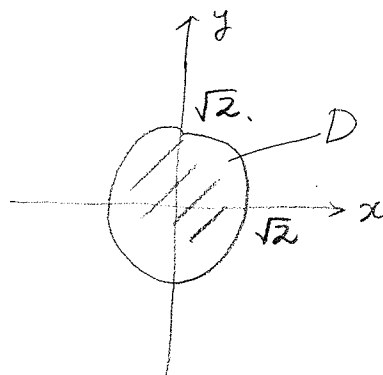
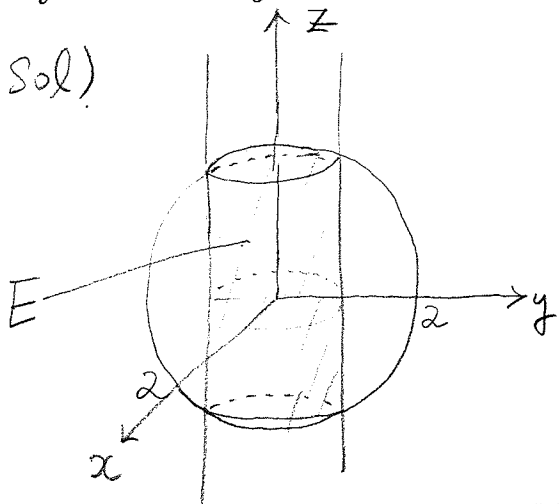


$$\int_{-1}^1 \int_{-3}^3 \int_0^{9-y^2} x e^y dz dy dx = \int_{-1}^1 x \int_{-3}^3 \int_0^{9-y^2} e^y dz dy dx =$$

$$\underbrace{\int_{-1}^1 x dx}_{=0} \int_{-3}^3 \int_0^{9-y^2} e^y dz dy = 0.$$

**Question 2:** [5pt] Find the volume of the solid that lies within both the cylinder  $x^2 + y^2 = 2$  and the sphere  $x^2 + y^2 + z^2 = 4$ .

Sol.)



$$V = \iiint_E dV = \iint_D \int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} dz dA = \int_0^{2\pi} \int_0^{\sqrt{2}} \int_{-\sqrt{4-r^2}}^{\sqrt{4-r^2}} r dz dr d\theta$$

$$= \int_0^{2\pi} \int_0^{\sqrt{2}} 2r\sqrt{4-r^2} dr d\theta = 2\pi \left[ \frac{2}{3} (4-r^2)^{\frac{3}{2}} (-1) \right]_0^{\sqrt{2}} =$$

$$-\frac{4\pi}{3} \left( 2^{\frac{3}{2}} - 4^{\frac{3}{2}} \right) = \frac{4\pi}{3} (8 - 2\sqrt{2}).$$