Math241, Quiz 7-version a, Nov. 5

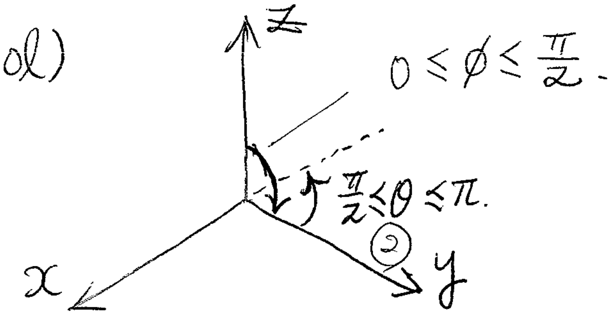
Name: Solutions

Question 1: **[5pt]** Evaluate the following integral.

$$\iiint_E e^{\sqrt{x^2+y^2+z^2}}\,dV,$$

where E is bounded by the sphere $x^2+y^2+z^2=4$ in the second octant.

Sol)



(ρ, θ, ϕ)

$\rho^2 = x^2 + y^2 + z^2$

$$\int_0^{\frac{\pi}{2}}\int_{\frac{\pi}{2}}^{\pi}\int_0^{2} e^{\rho}\rho^2\sin\phi\,d\rho\,d\theta\,d\phi = \frac{\pi}{2}\int_0^{\frac{\pi}{2}}\sin\phi\int_0^2 \rho^2 e^{\rho}\,d\rho\,d\phi =$$

$$\frac{\pi}{2}\underbrace{\int_0^{\frac{\pi}{2}}\sin\phi\,d\phi}\;\underbrace{\int_0^2 \rho^2 e^{\rho}\,d\rho} = \frac{\pi}{2}\cdot 1\cdot 2(e^2-1) = \boxed{\pi(e^2-1)}.$$

$\left[-\cos\phi\right]_0^{\frac{\pi}{2}}$

$= 1.$

$\left\{\begin{array}{ll} u=\rho^2 & dv = e^{\rho}d\rho \\ du = 2\rho\,d\rho & v = e^{\rho} \end{array}\right\}$

$$\left[\rho^2 e^{\rho}\right]_0^2 - 2\underbrace{\int_0^2 \rho e^{\rho}\,d\rho}$$

$\left\{\begin{array}{ll} u=\rho & dv = e^{\rho}d\rho \\ du = d\rho & v = e^{\rho} \end{array}\right\}$

$$= 4e^2 - 2\left(\left[\rho e^{\rho}\right]_0^2 - \int_0^2 e^{\rho}\,d\rho\right) = 4e^2 - 2\left(2e^2 - \left[e^{\rho}\right]_0^2\right)$$

$$= 2(e^2-1).$$

Question 2: [5pt] Use the given ~~translation~~ transformation to evaluate the integral.

$$\iint_R (2x+4y)\,dA,$$

where R is the parallelogram with vertices (2,6), (0,2), (-2,-6), and (0,-2) in the xy-plane;

$$x = u + v, \quad y = 2u + 4v.$$

Sol) $\dfrac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} 1 & 1 \\ 2 & 4 \end{vmatrix} = 4 - 2 = 2.$

For the integral domain S in the uv-plane,

$u = x - v, \quad y = 2(x-v) + 4v, \quad y = 2x + 2v, \quad v = \frac{1}{2}y - x$

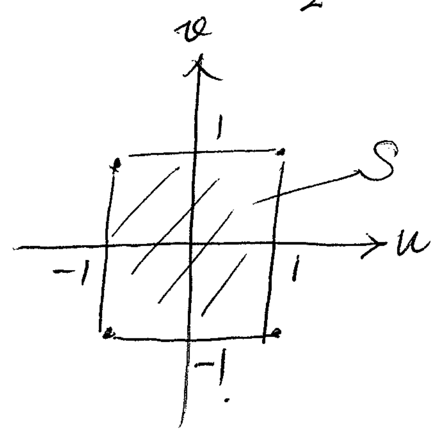
$u = x - \left(\frac{1}{2}y - x\right) = 2x - \frac{1}{2}y.$

$(2,6) \to (4-3, 3-2) = (1,1)$

$(0,2) \to (-1,1)$

$(-2,-6) \to (-4+3, -3+2) = (-1,-1)$

$(0,-2) \to (1,-1)$



$$\int_{-1}^{1}\int_{-1}^{1} \left(2(u+v) + 4(2u+4v)\right)\cdot 2\, du\,dv = 2\int_{-1}^{1}\int_{-1}^{1} (\underbrace{10u}_{\text{odd function. } -1 \le u \le 1} + 18v)\,du\,dv$$

$$= 2\int_{-1}^{1} \left[18vu\right]_{u=-1}^{1} dv = 36\int_{-1}^{1} \underbrace{2v}_{\text{odd function } -1 \le u \le 1.}\,dv = \boxed{0}$$