Math241, Quiz 8-version a, Nov. 12

Name: Solutions.

Question 1: **[5pt]** Evaluate the line integral, where C is the given curve.

$$\int_C ysinxdy,$$

C is the arc of the curve $y = sin(x)$ from $(0,0)$ to $(2\pi, 0)$.

Sol) $x = t$, $y = \sin(t)$, $0 \leq t \leq 2\pi$,

$$\int_0^{2\pi} \sin(t)\, \sin(t)\, \cos(t)\, dt$$

$$= \int_0^{2\pi} \sin^2(t) \cos(t)\, dt = \left[\frac{\sin^3(t)}{3} \right]_0^{2\pi} = \boxed{0}.$$

Question 2: [5pt] Evaluate the line integral $\int_C \vec{F} \cdot d\vec{r}$, where C is given by the vector function $\vec{r}(t)$.

$$\vec{F}(x, y, z) = x^2\vec{i} + (x + y)\vec{j} + z^2\vec{k},$$

$$\vec{r}(t) = t^2\vec{i} + t^4\vec{j} - t^2\vec{k}, 0 \leq t \leq 2.$$

Sol) $\vec{F}(\vec{r}(t)) = t^4\vec{i} + (t^2 + t^4)\vec{j} + t^4\vec{k}$,

$\vec{r}'(t) = 2t\vec{i} + 4t^3\vec{j} - 2t\vec{k}$,

$$\int_0^2 \cancel{2t^5} + (t^2 + t^4)4t^3 \cancel{-2t^5}\, dt =$$

$$4\int_0^2 t^5 + t^7\, dt = 4\left[\frac{t^6}{6} + \frac{t^8}{8}\right]_0^2 = 4\left(\frac{2^6}{6} + \frac{2^8}{8}\right) =$$

$$4\left(\frac{2^5}{3} + 2^5\right) = \frac{2^7}{3} + 2^7 = \frac{2^9}{3} = \boxed{\frac{512}{3}}.$$