Math241, Quiz 8-version b, Nov. 12

Name: Solutions.

Question 1: **[5pt]** Evaluate the line integral, where C is the given curve.

$$\int_C x sin y dx,$$

C is the arc of the curve $x = cos(y)$ from $(0, \frac{-\pi}{2})$ to $(0, \frac{\pi}{2})$.

Sol) $y = t$, $x = \cos(t)$, $-\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$,

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos(t)\sin(t)(-\sin(t)\,dt) =$$

$$-\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2(t)\cos(t)\,dt = -\left[\frac{\sin^3(t)}{3}\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} =$$

$$-\frac{1}{3}(1-(-1)) = \boxed{-\frac{2}{3}}.$$

Question 2: [5pt] Evaluate the line integral $\int_C \vec{F} \cdot d\vec{r}$, where C is given by the vector function $\vec{r}(t)$.

$$\vec{F}(x,y,z) = x^2\,\vec{i} + (x+y)\,\vec{j} + z^2\,\vec{k},$$

$$\vec{r}(t) = t^{\frac{3}{2}}\,\vec{i} + t^2\,\vec{j} + t^{\frac{3}{2}}\,\vec{k},\ 0 \le t \le 2.$$

Sol) $\vec{F}(\vec{r}(t)) = t^3\vec{i} + (t^{\frac{3}{2}} + t^2)\vec{j} + t^3\vec{k}$,

$\vec{r}\,'(t) = \frac{3}{2}t^{\frac{1}{2}}\vec{i} + 2t\,\vec{j} + \frac{3}{2}t^{\frac{1}{2}}\vec{k}$,

$$\int_0^2 \frac{3}{2}t^{\frac{7}{2}} + 2t(t^{\frac{3}{2}} + t^2) + \frac{3}{2}t^{\frac{7}{2}}\,dt =$$

$$\int_0^2 3t^{\frac{7}{2}} + 2t^{\frac{5}{2}} + 2t^3\,dt = \left[\cancel{3}\cdot\frac{2}{\cancel{9}_{3}}t^{\frac{9}{2}} + 2\cdot\frac{2}{7}t^{\frac{7}{2}} + \cancel{2}\cdot\frac{t^4}{\cancel{4}_{2}}\right]_0^2$$

$$= \frac{2}{3}2^{\frac{9}{2}} + \frac{4}{7}2^{\frac{7}{2}} + \frac{2^4}{2} = \frac{1}{3}2^{\frac{11}{2}} + \frac{1}{7}\cdot 2^{\frac{11}{2}} + 8 = \frac{10}{21}32\sqrt{2} + 8 =$$

$$\boxed{8 + \frac{320\sqrt{2}}{21}}$$