Complex number comprehension checks:

Make sure you understand (that you can justify) the following identities. To make algebra with complex numbers easier, note that you can always treat i as an unknown.

- $i^2 = -1$
- $i^3 = -i$
- $i^4 = 1$
- $\frac{1}{i} = -i$
- $(3-7i)(-2-9i) = \cdots = -69 - 13i$
- $(3-2i)(3+2i) = 3^2 - (2i)^2 = 3^2 + 2^2 = 13$

Complex division:

$$\frac{1}{3-2i} = \frac{1}{3-2i}\frac{3+2i}{3+2i} = \frac{3+2i}{13} = \frac{3}{13} + \frac{2}{13}i$$

Euler's formula:

$$e^{i\theta} = \cos\theta + i\sin\theta \qquad \text{and} \qquad e^{-i\theta} = \cos\theta - i\sin\theta$$

Exercise: Check the identities:

$\cos\theta = \dfrac{e^{i\theta} + e^{-i\theta}}{2}$

$\sin\theta = \dfrac{e^{i\theta} - e^{-i\theta}}{2i}$

Exercise: Double angle identities:

Start with $e^{i(2\theta)} = \left(e^{i\theta}\right)^2$. Use Euler on each side and deduce:

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$\sin 2\theta = 2\sin\theta\cos\theta$

Notation to remember:

Real part: $\mathrm{Re}(a+bi) = a$

Imaginary part: $\mathrm{Im}(a+bi) = b$