

Basic Matrix Operations — in-class worksheet

$$A = \begin{bmatrix} 2 & 1 \\ -1 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 3 & -2 \\ 7 & 1 \end{bmatrix} \quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

A is a 2×2 matrix, with entries:

$$\begin{aligned} a_{11} &= & a_{12} &= \\ a_{21} &= & a_{22} &= \end{aligned}$$

Compute:

$$A + B = \begin{bmatrix} & \\ & \end{bmatrix} = B + A, \quad 3A = \begin{bmatrix} & \\ & \end{bmatrix}$$

The column vectors of A are:

$$\begin{bmatrix} & \\ & \end{bmatrix}, \begin{bmatrix} & \\ & \end{bmatrix}$$

The transpose of A is

$$A^T = \begin{bmatrix} & \\ & \end{bmatrix}$$

The products AB and BA are not equal:

$$AB = \begin{bmatrix} & \\ & \end{bmatrix}, \quad BA = \begin{bmatrix} & \\ & \end{bmatrix}$$

A times a vector gives a linear combination of the columns of A :

$$A \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} & \\ & \end{bmatrix} = c_1 \begin{bmatrix} 2 \\ -1 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

Multiplying by the identity leaves a matrix unchanged:

$$AI = \begin{bmatrix} & \\ & \end{bmatrix} = A, \quad IA = \begin{bmatrix} & \\ & \end{bmatrix} = A$$

The inverse of A is

$$A^{-1} = \begin{bmatrix} & \\ & \end{bmatrix}$$

Check:

$$A^{-1}A = \begin{bmatrix} & \\ & \end{bmatrix} = I, \quad AA^{-1} = \begin{bmatrix} & \\ & \end{bmatrix} = I$$

To compute the determinant of the 3×3 matrix

$$C = \begin{bmatrix} 2 & 4 & -1 \\ 3 & 1 & 4 \\ -2 & 5 & 1 \end{bmatrix}$$

we first find the three submatrices

$$C_{11} = \begin{bmatrix} & \\ & \end{bmatrix}, \quad C_{12} = \begin{bmatrix} 3 & 4 \\ -2 & 1 \end{bmatrix}, \quad C_{13} = \begin{bmatrix} & \\ & \end{bmatrix},$$

and then use the cofactor expansion across the first row:

$$\det(C) = 2 \det(C_{11}) - 4 \det(C_{12}) + (-1) \det(C_{13}) =$$