

## Example of Forced Vibrations

Here we have the equation for a vibrating string with  $a = 1$ , length is 1, and we are forcing with force 1 and frequency 1. That is we have the system

$$\begin{aligned} y_{tt} &= y_{xx} + \cos t \\ y(0, t) &= y(1, t) = 0 \\ y(x, 0) &= y_t(x, 0) = 0 \end{aligned}$$

We got a particular solution  $(\cos x - \frac{\cos 1 - 1}{\sin 1} \sin x - 1) \cos t$  and we took the odd 2 periodic extension of the function  $\cos x - \frac{\cos 1 - 1}{\sin 1} \sin x - 1$  (this was defined on  $0 < x < 1$ ) and we call the extension  $F$ . Then the solution was

$$y(x, t) = \frac{F(x+t) + F(x-t)}{2} + (\cos x - \frac{\cos 1 - 1}{\sin 1} \sin x - 1) \cos t$$

Which has the following plot.

