Math 380, Practice Exam 1

These problems may (or may not) be slightly longer / harder than real test problems.

1. Which points (circle all) are on the line described by $x-y-2z=3$ and $2x-y+3z=-6$?

   A. $(1,1,1)$ B. $(2,1,-1)$ C. $(1,-2,-2)$ D. $(3,0,0)$

2. Let $u = x^3 - y^2$ and $v = xy^2$.
   (a) Find the Jacobian matrix of the mapping
   (b) Find the Jacobian matrix of the inverse mapping
   (c) Find $\frac{\partial^2 x}{\partial u \partial v}$

3. Let $e^{x^2-y^2} - e^{u+v} = 0$ and $e^{2xy} - e^{u-v} = 0$.
   (a) Compute $\left(\frac{\partial u}{\partial x}\right)_y$
   (b) At what points can we find (u,v) as a vector valued function of x and y using the implicit function theorem. That is, exactly at what points can we apply the procedure in section 2.10 of the book.

4. Let $x = t^3$, $y = t^5$, and $z = t^7$ define a curve.
   (a) Why is the point $(1,1,1)$ on the curve?
   (b) Find a vector of magnitude 1 tangent to the curve at the point $(1,1,1)$.
   (c) Find the equation of the plane normal to the curve at the point $(1,1,1)$.

5. Let $f(x,y) = \sin(x^2 - y^3)$.
   (a) Compute ∇f
   (b) Compute $\nabla^2 f$

6. Let a vector field be given by $\mathbf{v} = x^2\mathbf{i} + xyz\mathbf{j} + z^2\mathbf{k}$.
   (a) Compute $\operatorname{div} \mathbf{v}$
   (b) Compute $\operatorname{grad} \operatorname{div} \mathbf{v}$
   (c) Compute $\operatorname{curl} \mathbf{v}$

7. Let $z = (x-y)^{3000}$.

   Compute $\frac{\partial^2 z}{\partial x \partial y}$.

8. Find critical points and classify if they are maxima or minima for the function $z = 2x^2 + 4y^2$ subject to $x^4 + y^4 = 1$.