Math 380, Practice Exam 2

Covered sections are 3.3–3.6, 4.3–4.7, 4.9, 5.1–5.8. Knowledge of previous sections is required inasmuch it will be needed for problems from the covered sections. These problems may (or may not) be longer / harder than real test problems.

1. Use a line integral to find a closed form function $F(x, y)$ where $\frac{\partial F}{\partial x} = 2xy$, $\frac{\partial F}{\partial y} = x^2 - y^2$ and $F(0, 0) = 0$.

2. Let R be a circle centered at $(0, 0)$ and of radius 10 in the plane.

   Compute: $\iint_R e^{x^2+y^2}\, dxdy$

3. Let C be the curve in the plane parametrized by $x = \cos t(1-\cos t)$ and $y = \sin t(1-\cos t)$ for $0 \le t \le 2\pi$.

   Compute: $\oint_C x^3 dx + y^7 dy$

4. Let C be the curve $y = x^2$ for $0 \le x \le 1$, oriented in the direction of increasing x.

   Let $\mathbf{v} = 2xy\mathbf{i} + (x^2 + y^2)\mathbf{j}$. Let v_n be the normal component of $\mathbf{v}$, where the normal is $90°$ behind the tangent as usual. Let v_T be the tangential component of $\mathbf{v}$.

   (a) Compute: $\underset{C}{\int_{(0,0)}^{(1,1)}} v_n\, ds$

   (b) Compute: $\underset{C}{\int_{(0,0)}^{(1,1)}} v_T\, ds$

5. Set up but do not compute the integral for computing the surface area of the following surface in $\mathbb{R}^3$ in parametrized form (i.e. the surface is parametrized by u and v).

   $x = 5u + v^3$, $y = u^3 - 5v$, $z = e^{u^2-v^2}$, for $-1 \le u \le 1$ and $-1 \le v \le 1$.

6. Suppose that a cube of side length 1 meter has density equal to $1 + r^2 \frac{Kg}{m^3}$, where r is the distance from one corner.

   (a) Compute the mass of the cube.

   (b) Pick coordinates x, y and z such that the cube is given by $0 \le x, y, z \le 1$ and r is the distance from $(0, 0, 0)$. Compute the center of mass of this cube in these coordinates.

7. Define the vector field $\mathbf{v} = \nabla F + x\mathbf{i}$, where $F(x, y, z) = \sin(x)e^{y+z}$.

   Compute: $\operatorname{curl} \mathbf{v}$

8. Suppose $\mathbf{v}$ is a vectorfield such that $\operatorname{div} \mathbf{v} = x^2 + y^2$. Let C be the unit circle, and let $\mathbf{n}$ be the outer unit normal.

   Compute: $\oint_C \mathbf{v} \cdot \mathbf{n}\, ds$