Math 380, Practice Exam 3

Covered sections are 5.9 – 5.16 (for 5.14 – 5.16 only as much as was covered in class). Knowledge of previous sections is required inasmuch it will be needed for problems from the covered sections. These problems may (or may not) be longer / harder than real test problems.

1. Let the curve C be given by $x = t^3$, $y = t^2$, $z = -t$, for $-1 \leq t \leq 1$ oriented in the direction of increasing t.

   Compute: $\displaystyle\int_C xy\ dx + xyz\ dy + z^2x\ dz$

2. Let S be the surface given by $z = 4 - x^2 - y^2$ and $x^2 + y^2 \leq 4$. Suppose the normal $\mathbf{n}$ is the upper unit normal. Let $\mathbf{u} = xz\mathbf{i} + x^2\mathbf{j} + xyz\mathbf{k}$.

   Compute: $\displaystyle\iint_S \mathbf{u} \cdot \mathbf{n}\ d\sigma$

3. Let S be the surface of the unit sphere, $x^2 + y^2 + z^2 = 1$. Let $\mathbf{n}$ be the outer normal. Compute

   Compute: $\displaystyle\iint_S x^2z\ dy\ dz + x^7z^5\ dz\ dx - xz^2\ dx\ dy$

4. Let S be the surface of the unit sphere, $x^2 + y^2 + z^2 = 1$. Let $\mathbf{n}$ be the outer normal. Let $F(x, y, z) = x^2 + y^2 - z$.

   Compute: $\displaystyle\iint_S \nabla F \cdot \mathbf{n}\ d\sigma$

5. Let C be the closed curve given by $x = \cos(t)$, $y = \sin(t)$, $z = \cos^2(t)$, $0 \leq t \leq 2\pi$ oriented in the direction of increasing t. Let $F(x, y, z) = e^{x^2+x-z}$

   Compute: $\displaystyle\int_C (\nabla F)_T\ ds$

6. Let C be the curve given by $z = \sin(2\pi x)$, $y = \cos(2\pi x)$, for $0 \leq x \leq 1$, oriented in the direction of positive x.

   Compute: $\displaystyle\int_C 2xye^z\ dx + x^2e^z\ dy + x^2ye^z\ dz$