Math 380, Practice Final

These problems may (or may not) be longer / harder than real test problems. These are just extra problems, use practice exams 1-3 and the real exams 1-3 for practice.

1. Let $x = \cos t$, $y = \sin t$, and $z = t^3$ define a curve.
   (a) Find the tangent line to the curve at the point corresponding to $t = \frac{\pi}{2}$.
   (b) Find the equation for the plane normal to the curve at the point corresponding to $t = \frac{\pi}{2}$.

2. Suppose a cylinder of radius 1 meter and length 10 meters has density $(1+d^2)\ \frac{kg}{m^3}$, where d is the distance from one end of the cylinder.
   (a) Set up (but do not yet compute) a triple iterated integral for the mass of the cylinder.
   (b) Compute the mass of the cylinder in kg.
   Hint: use cylindrical coordinates

3. Let $f(x) = \displaystyle\int_{\sin(x)}^{\cos(x)} e^t x \, dt$

   Compute $f'(x)$:

4. Let C be the curve given by $z = \sin(2\pi x)$, $y = \cos(2\pi x)$, for $0 \le x \le 1$, oriented in the direction of positive x.

   Compute: $\displaystyle\int_C 2xye^z \, dx + x^2 e^z \, dy + x^2 y e^z \, dz$

5. Let C be the closed curve given by $x = \cos(t)$, $y = \sin(t)$, $z = \cos^2(t)$, $0 \le t \le 2\pi$ oriented in the direction of increasing t. Let $F(x, y, z) = e^{x^2+x-z}$

   Compute: $\displaystyle\int_C \nabla F \, ds$

6. Let S be the surface $x^2 + y^2 = 1$ and $0 \le z \le 1$ (just the side of the cylinder not the caps on the top and bottom). Let $\mathbf{v} = xyz\mathbf{i} + e^{x^2+y^2+z^2}\mathbf{j} + (z+1)(x^2+y^2)\mathbf{k}$ and let $\mathbf{n}$ be the outer normal.

   Compute $\displaystyle\iint_S \operatorname{curl} \mathbf{v} \cdot \mathbf{n} \, d\sigma$

7. Find the equation for the plane that best approximates $z = x^3 + 3xy + y^3$ at the point where $x = 1$ and $y = 2$.

8. Let $\psi = \log \frac{1}{\sqrt{x^2+y^2}}$ be the potential function in a plane and let $\mathbf{F} = \nabla\psi$ be the corresponding force field. Let C be the unit circle $x^2 + y^2 = 1$. Let $\mathbf{n}$ be the unit outer normal.
   (a) Compute $\displaystyle\oint_C F_n \, ds = \oint_C \mathbf{F} \cdot \mathbf{n} \, ds$
   (b) Compute $\displaystyle\oint_C F_T \, ds$