

HW1, DUE AUG 31.

1.1: 3) Prove

- a) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
- b) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

1.1: 4) Let $A \Delta B$ denote the *symmetric difference*, that is, the set of all elements that belong to either A or B , but not to both A and B . Draw a Venn diagram for $A \Delta B$.

- a) Show $A \Delta B = (A \setminus B) \cup (B \setminus A)$.
- b) Show $A \Delta B = (A \cup B) \setminus (A \cap B)$.

1.1: 5) For each $n \in \mathbb{N}$ let $A_n := \{(n+1)k \mid k \in \mathbb{N}\}$.

- a) Find $A_1 \cap A_2$.
- b) Find $\bigcup_{n=1}^{\infty} A_n$ and $\bigcap_{n=1}^{\infty} A_n$.

1.1: 13) Suppose that $f: A \rightarrow B$ and $G, H \subset B$. Show that $f^{-1}(G \cup H) = f^{-1}(G) \cup f^{-1}(H)$ and $f^{-1}(G \cap H) = f^{-1}(G) \cap f^{-1}(H)$.

1.1: 16) Let $a, b \in \mathbb{R}$ with $a < b$. Find an explicit bijection of $A := \{x: a < x < b\}$ and $B := \{y: 0 < y < 1\}$.

1.1: 20) Let $f: A \rightarrow B$ and $g: B \rightarrow C$ be functions.

- a) Prove that if $g \circ f$ is injective, then f is injective.
- b) Prove that if $g \circ f$ is surjective, then g is surjective.

1.2: 1) Prove $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \cdots + \frac{1}{n(n+1)} = \frac{n}{n+1}$ for all $n \in \mathbb{N}$.

1.2: 2) Prove $1^3 + 2^3 + \cdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$ for all $n \in \mathbb{N}$.

1.2: 6) Prove that $n^3 + 5n$ is divisible by 6 for all $n \in \mathbb{N}$.

1.2: 16) Find all $n \in \mathbb{N}$ such that $n^2 < 2^n$.