Mathematical thinking is more than memorizing theorems and working problems. As mathematicians, we make conjectures and test them; we look for patterns; we connect new ideas to familiar ones; we generalize, abstract and describe. Yet all too often, I believe we short-change our students by simply presenting theorems and examples from a book instead of incorporating them in the world of mathematical thought. A teacher may give a lecture that is detailed, concise and elegant, but if the students are not engaged and thinking mathematically, I believe we miss the real goal of teaching mathematics. Not everyone will become a mathematician in the strictest sense, but everyone can benefit from learning to think mathematically.

Where appropriate, I employ technology to motivate my students and help them thinking mathematically. In three semesters I have taught a class in the Calculus and Mathematica (C&M) sequence at the University of Illinois. In this format, traditional lectures are replaced by step-by-step tutorials, animations and in-depth problems worked on the computer. During class the students are given attention individually or in groups as necessary. In this way, students not only engage with technology but also with each other as they assimilate new material. I believe that mathematics is not a "spectator sport" and that students in the C&M classes benefit by doing the math rather than just seeing the concepts presented. For example, in my Vector Calculus with Mathematica class, a pair of students were given the task of finding the closest point along a line to another point not on the line. They chose to minimize a distance formula by taking a derivative, but when I asked them to plot a vector from the given point to this new point, they instantly noticed a perpendicular angle forming. We then had the chance to talk about other methods for solving the problem, like solving an equation involving the dot product or using a projection formula, and the advantages of each. This enabled them to think mathematically. I've found that students make these kind of connections often when the appropriate technology is at hand.

During another semester I taught Elementary Linear Algebra. We covered a number of methods for solving systems of linear equations, such as by finding the reduced row echelon form (RREF) of the corresponding matrix. But when the number of equations equals the number of variables, one can also write the system as a matrix equation $A\mathbf{x} = \mathfrak{b}$ and multiply both sides of the equation by A^{-1} (assuming it exists), or one could use Cramer's Rule (again, assuming $\det(A) \neq 0$). After having been introduced to all three methods, one of my students asked, "Why do we care about the new methods if we could always just take the RREF of the matrix?" This is an excellent mathematical question which led to a discussion of the benefits of the different methods. For instance, if you know that the entries of A^{-1} and $\mathfrak{b}$ are positive, then the entries of $A^{-1}\mathfrak{b}$ are also positive, which is hard to see using the other two methods. (Problems of this type come up when considering Leontief's economic model.) The lesson for me was that *why* we learn something is just as important as *how* we do it. Discussing the motivation for different methods gives students the justification for why we teach what we do and the judgement for when to use different methods. Again, this creates mathematically thinking students.

Regardless of the format of the class, communication is an important component of mathematical thinking. With that in mind, I value student feedback to help

me improve my teaching ability. Student end-of-semester feedback forms indicate aspects of my teaching that worked well and ideas to try in the future. Feedback has been positive: I've been on the List of Teachers Ranked as Excellent by their Students 4 different semesters. (Selected student comments appear below.) More recently I was awarded the Brahan TA Instructional Award for Teaching. I also have students fill out informal feedback forms early in the semester to gauge how the class is going and give the students a chance to voice concerns. During my C&M Calculus 1 course, many students commented that they would like to see more problems worked out during discussion, so I altered my presentations to allow more time for examples. Thus, I benefited by learning how to tailor my presentations to my students, and my students benefited by influencing the style of the course.

I'm constantly looking for new ways to get my students thinking mathematically. Sometimes this includes talking to other professors, graduate students or former students. I have also taken education classes in pursuit of a Teaching Masters Degree, which I completed in December 2008. I've read about, discussed and tried alternative teaching methods, especially those relating to technology. I hope to continue to improve my teaching so future students can appreciate the utility of mathematical thought.

Selected Student Comments

(complete responses available upon request):

- "You did a very good job of teaching material of the subject. I feel like I learned everything about the subject from you." (Math 220 - Fall 2006)
- "Fantastic Job! Best TA of my college career and I'm a senior! I was so afraid of this course and you made everything clear and even interesting." (C&M Math 220 - Spring 2007)
- "Amazing at explanation." (C&M Math 220 - Spring 2007)
- "You as a TA are fantastic. No matter how confused I may be, you are always able to simplify or explain something in a way that makes it more understandable." (C&M Math 220 - Spring 2007)
- "Jason seemed like he really did know the material we were learning and also that he cared about us learning it and making sure we understood it." (C&M Math 241 - Spring 2008)
- "The instructor was very well prepared for lectures and knew the subject and mathematica program very well to help the students whenever there was a problem." (C&M Math 241 - Spring 2008)
- "Great thorough explanations. Strong understanding of course materials." (C&M Math 241 - Fall 2008)