HW3, M588, 2008 SPRING
DUE: March 10 (M) by 11:59:00am. From 11:59:01am it counts as a late homework) NAME:..............................

SCORE:.............

For exercises 1,2, 3, 5 and 6 **NO collaborative work is allowed**! Please write each solution to a separate (new) page.

1. Given the following LP, which is called as primal P:

$$\min x_1 + x_3$$

$$x_1 + 2x_2 \leq 5$$

$$x_2 + 2x_3 = 6$$

$$x_1, x_2, x_3 \geq 0.$$

(a) Solve P by the simplex algorithm.
(b) Write the dual D of P by inspection.
(c) Write the complementary slackness condition for this problem and use them to solve the dual D. Check your answer by evaulating the optimal costs of P and D.

2. Give a complete proof (Theorem 7.1 can be used) for the following theorem: (copying from the book does not count as a complete solution!):
Let f_1 be an optimal flow of value v in an instance of min-cost flow. Let f_2 be a flow of value 1 along an $s-t$ augmentation path P in $N'(f_1)$ of least cost. Then $f_1 + f_2$ is an optimal flow of value $v+1$.

3. Given the following network with edges $[sa, sb, ac, ba, bc, cb, ct, bt]$, capacities $[2, 4, 3, 3, 1, 2, 4, 3]$, costs $[2, 3, 1, 2, 2, 1, 1, 5]$. The aim is to have a min-cost flow with value 5.
(i) Write this problems as a dual in standard form.
(ii) Given the following flow, construct the incremental weighted flow network N', and write the RP of this problem: The flowvector: $[2, 3, 3, 1, 1, 1, 2, 3]$.
(iii) Write down the DRP problem. Solve the DRP problem (you can just provide the solution, you do not have to show your work).
(iv) Given the solution for the DRP, improve the solution for the D, show the improved flow.

4. Prove that there is an optimal solution to the LP for the shortest-path problem in which each $f_i = 0$ or 1.
When are there also optimal solutions which violate this condition?

5. Solve the following Hitchcock problem. Show your work:
Supplies: $a_1 = 3, a_2 = 2, a_3 = 1, a_4 = 4$. Demands: $b_1 = 1, b_2 = 3, b_3 = 2, b_4 = 2, b_5 = 2$. The cost-matrix is (listed row-wise): $[3, 2, 3, 1, 2]$, $[1, 5, 4, 5, 2]$, $[4, 4, 3, 2, 1]$, $[5, 1, 3, 5, 2]$.

6. Write (TYPE) a one-page summary about the ellipsoid method. Give an overview, and as much technical details as possible, but still should be readable, and some computation should be omitted.