Math 447 Homework #3 Answers

8.4 Suppose $\varepsilon > 0$. Since $s_n \to 0$, $\exists N \ni$ if $n > N$,

then $|s_n - 0| = |s_n| < \frac{\varepsilon}{M}$. Then for $n > N$ we have

$$|s_n t_n - 0| = |s_n t_n| \le M|s_n| < M \cdot \frac{\varepsilon}{M} = \varepsilon,$$

Thus $s_n t_n \to 0$.

8.10 Let $s = \lim_{n\to\infty} s_n$. We are given $s > a$.

We apply the definition of convergence with

$\varepsilon = \frac{s-a}{2} > 0$ to conclude $\exists N \ni$ if $n > N$ then

$$|s_n - s| < \varepsilon = \frac{s-a}{2}.$$

This is equivalent to

$$\frac{a-s}{2} < s_n - s < \frac{s-a}{2}$$

So, for $n > N$, we have $s_n > \frac{a-s}{2} + s = \frac{a+s}{2} > a$.